MEASUREMENT OF THE INFORMATION VELOCITY IN FAST- AND SLOW-LIGHT OPTICAL PULSE PROPAGATION

by

Michael David Stenner

Department of Physics
Duke University

Date:____________________________

Approved:

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______________________________
Dr. John E. Thomas

______________________________
Dr. Henry Everitt

______________________________
Dr. Henry S. Greenside

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Duke University

2004
ABSTRACT

(Physics)

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Abstract

This thesis describes a study of the velocity of information on optical pulses propagating through fast- and slow-light media. In fast- and slow-light media, the group velocity \( v_g \) is faster than the speed of light in vacuum \( c \) (\( v_g > c \) or \( v_g < 0 \)) or slower than \( c \) (\( 0 < v_g < c \)) respectively. While it is largely accepted that optical pulses can travel at these extreme group velocities, the velocity of information encoded on them is still the subject of considerable debate. There are many contradictory theories describing the velocity of information on optical pulses, but no accepted techniques for its experimental measurement. The velocity of information has broad implications for the principle of relativistic causality (which requires that information travels no faster than \( c \)) and for modern communications and computation.

In this thesis, a new technique for measuring the information velocity \( v_i \) is described and implemented for fast- and slow-light media. The fast- and slow-light media are generated using modern dispersion-tailoring techniques that use large atomic coherences to generate strong normal and anomalous dispersion. The information velocity in these media can then be measured using information-theoretic concepts by creating an alphabet of two distinct pulse symbols and transmitting the symbols through the media. By performing a detailed statistical analysis of the received information as a function of time, it is possible to calculate \( v_i \). This new technique makes it possible for the first time to measure the velocity of information on optical pulses.
Applying this technique to fast-light pulses, where $v_g/c = -0.051 \pm 0.002$, it is found that $v_i/c = 0.4(+0.7 - 0.2)$. In the slow-light case, where $v_g/c = 0.0097 \pm 0.0003$, information is found to propagate at $v_i/c = 0.6$. In the slow-light case, the error bars are slightly more complicated. The fast bound is $-0.5c$ (which is faster than positive values) and the slow bound is $0.2c$. These results represent the first measurements of the velocity of information on fast- and slow-light optical pulses and are inconsistent with theories that predict $v_i = v_g$ for slow-light pulses (or fast-light pulses) and are consistent with relativistic causality and a recent proposal that information always propagates at a velocity $v_i = c$. 
Acknowledgments

Graduate study is one of those strange pursuits that is intensely solitary and personal, but is simultaneously deeply intertwined with the rest of one’s life. My life and accomplishments as a graduate student have been deeply affected by the other events of my life and the people in it. I would therefore like to acknowledge those people who have helped me along my path.

I must begin with my parents, David and Ann Stenner. Neither of them are academics by nature, and I am something of an oddity in the family. As such, they have never really understood my quest, neither the scientific nature of my work, nor my own personal motivations for doing it. However, aside from the obligatory pre-college questions like “yes, but what will you do with a physics major?”, they have been unwaveringly supportive of my academic and professional choices. The fact that these choices certainly seemed strange to them only deepens my respect and gratitude.

My father also deserves my thanks for the many years of training he gave me; my role as “apprentice handyman” was better preparation for experimental physics than either of us could have imagined. He gave me a general familiarity with basic mechanical, electrical, plumbing and construction techniques that have aided me immensely. Even more important, he gave me the willingness and confidence to roll up my sleeves and dive into such tasks.

I am lucky to have many wonderful friends in my life who have shaped and
molded me by their very presence. I cannot possibly thank all of these friends here, but I would like to mention a few. My friends from before graduate school—Anna (Gander) Stephenson, Adam Thorne, Dave Harrington, Michael Donnelly, Chris Schmidt, to name a few—have truly influenced the man that I am and the way I approach life and the world around me. Graduate school is a special place, and so the friendships with fellow graduate students are of a special sort. I was very lucky to have a friendly, supportive, and talented incoming class. Coming into a new city, school, and concept of hard work, one’s classmates are extremely important. I would like to specifically thank Timothy Burt, Chee Liang Hoe, Linda Waters, and Mike Kirby; each of these dear friends has been a massive influence on me and has provided crucial support and friendship.

As the years have progressed and people have gone their separate ways, my collection of close friends has evolved. More recently I’ve come to eat, talk, and socialize regularly with “the lunch crowd”: Mike Gehm, Seth Vidal, and Konstantin “Icon” Riabitsev. Until he abandoned us for fame and fortune in Alabama, Stephen Granade was also in this list. One of these, Mike Gehm, was kind enough to serve double duty as close friend and strong professional influence; I’ve come to think of him as my “physics soul-mate”, a label supported by our strangely similar professional left-turns. Very recently, I have come to know my neighbors and friends Lisa Waldo and Lynn Eaton. They have been immensely supportive and helpful in this particularly difficult part of my life.

I must also thank the members of my research group, with which I couldn’t be more pleased. From my early days in this group, there have always been lively discussions of politics, religion, ethics, and a little more politics. This group has
always been (and continues to be) very diverse in nationality, gender, political views, and personality. The members of this group have also brought a broad range of technical skills and experiences, which have proven extremely useful on many occasions. In my first years here, I had the pleasure of working with Bill Brown on the two-photon laser project, and also with the post-docs Jeff Gardner, Olivier Pfister, and John Swartz on that project. After I moved to the information velocity experiments, I was able to share a lab and laser (if not an experiment) with Heejeong Jeong and Jean-Philippe Smits. Both of them are talented scientists and I thoroughly enjoyed working with them. They also provided me with one of my first experiences with scientific leadership, mentoring, and management. For this, I both thank them and apologize to them.

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As much as the life of a physics graduate student is focused on physics, there are nonetheless technical and bureaucratic tasks that must be done. I must therefore thank the systems staff, Seth Vidal and Konstantin Riabitsev for providing an
excellent computing infrastructure and technical assistance. On the bureaucratic front, Donna Ruger, Maxine Stern, and Shari Wynn made many daunting problems and tasks simply disappear, which is the highest compliment I can give them.

My committee members, Dr. John Thomas, Dr. Henry Everitt, and Dr. Henry Greenside provided many useful comments and suggestions, helping dramatically improve the quality and clarity of this dissertation. Also, John Thomas often felt like a second advisor with his always-open door and unbridled enthusiasm. Dr. Mark Neifeld would have been on my committee were it not for scheduling restrictions. His technical assistance in the last several months of this project have not just been valuable; without them, I could not have done this work. Furthermore, he performed all of the responsibilities of a committee member, carefully reading and commenting on my dissertation. Therefore, I thank him as well, as an honorary member of my committee.

The most important member of my committee is of course my advisor and mentor, Dr. Daniel Gauthier. From him, I have learned an enormous amount of science. More importantly, I have learned an enormous amount about being a scientist. He is an incredible scientist with a strong sense of professional responsibility and integrity. It has been an honor to work with him and I would welcome the opportunity to collaborate with him in the future.

The last person I must thank is also the most difficult one to thank properly. Annie has been many things to me throughout my graduate study here; for most of it, she was my wife and partner. Although that has changed now, she remains—as she has always been—my dear and loyal friend. Her unwavering support, confidence, and pride have been invaluable to me, and are deeply appreciated.
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Chapter 1

Introduction

By all accounts, modern science and engineering has a good understanding of how to use pulses of light to communicate information. It is, after all, the basis for one of the world’s biggest and fastest-growing industries. And yet, the fundamental question of how fast information travels remains unanswered. The engineering community, starting with the seminal work by Shannon [1], has studied information rates, but has essentially ignored the question of the velocity of information $v_i$. The physics community, initially prompted by an apparent challenge to Einstein’s special theory of relativity [2], has been debating the issue off and on for almost 100 years. Surprisingly, the issue remains unresolved. There is no clear definition of the information velocity because there is only a vague understanding of where information is contained on a waveform. In this thesis, I present a technique for experimentally measuring the velocity of information $v_i$ and apply this technique to study optical pulse propagation in media with “fast” and “slow” group velocities.

1.1 Optical pulse propagation in dispersive media

Most people think of optical pulses as discrete objects with easily defined velocities. Solid discrete objects are indeed easy to characterize. For example, it is easy to
describe the velocity of a bullet as it moves through the air because it is a single solid object and remains unchanged as moves. But what about a bullet traveling through something more substantial, like wood? The bullet may become distorted as it travels through the wood; it may flatten, or it may break up into bits. How, then, do we describe its velocity? We can certainly define a number of characteristic velocities that may be useful—the velocity of the foremost part of the bullet, the velocity of the center of mass, etc.—but to call any of these “the velocity of the bullet” would be an oversimplification at best. In some ways, optical pulses are like bullets; they can propagate unchanged in vacuum, but they change their shape or propagate strangely when traveling through a dispersive medium.

The reason optical pulses do not propagate as discrete objects in dispersive media is because all pulses are composed of multiple frequencies, which can each behave differently. Basic Fourier theory tells us that almost any function of time can be represented as a function of frequency and vice versa. Furthermore, the temporal and spectral characters of pulses are specifically related by the Fourier transform. Any nontrivial function of time is necessarily composed of multiple frequencies. If a pulse is narrow in time (i.e., short), it must be wide in frequency. If a pulse consists of only a finite range of frequencies, then it must exist for all times! Some examples of pulses and their spectra are shown in Fig. 1.1.

This multi-frequency nature of pulses becomes very important when the pulses propagate through a dispersive medium. In any medium, the propagation of a single frequency is governed by the refractive index $n$. However, in dispersive media, the refractive index varies with frequency. The result of this variation is that each
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Figure 1.1: Sample pulses and their spectra. (a) shows the intensity of the pulses as a function of time. (b) shows the intensity as a function frequency. Three pulses are shown: a Gaussian (solid), a Lorentzian (dotted), and a square pulse (dashed).

individual frequency component propagates with its own phase velocity

\[ v_p \equiv \frac{c}{n(\omega)}, \]  

(1.1)

where \( \omega \) is the frequency of the individual component and \( c \) is the speed of light in vacuum. The phase velocity is the velocity of a peak in an oscillating monochromatic electromagnetic field, or the velocity of a point of constant phase. Because different frequencies have different phase velocities in dispersive media, the pulse as a whole does not necessarily retain its shape. In general, a pulse can become very distorted when this occurs.

If the dispersion—the variation in \( n(\omega) \)—is nearly linear, the pulse may largely retain its shape over finite distances but move at a velocity different from \( v_p \), called the group velocity, and given by

\[ v_g \equiv \frac{c}{n(\omega) + \omega \frac{dn}{d\omega}} \bigg|_{\omega = \omega_0}. \]  

(1.2)
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Here, $\omega_0$ is the central frequency of the pulse. The pulse will propagate mostly undistorted at the group velocity if (1) the refractive index varies linearly over a range of frequencies and (2) the spectrum of the pulse falls mostly within this frequency range. The better these conditions are met, the less distortion there will be in the pulse. However, these conditions can never be met perfectly, and so $v_g$ is often called “the group velocity approximation” [3-6].

A pulse’s group velocity can be faster or slower than $c$. This is what is meant by the terms “fast” and “slow”; when a pulse’s group velocity is faster than $c$, the pulse is referred to as “fast,” and when $v_g$ is slower than $c$, the pulse is “slow.” It is possible to create fast and slow light in non-dispersive (or weakly dispersive) media where $dn/d\omega \approx 0$. In this case, $v_g \approx v_p$ and the group velocity is completely determined by the index $n$. However, it is often more convenient to engineer systems with large dispersion so that $dn/d\omega$ has very large positive or negative values. The creation of these systems is discussed below in Ch. 1.3.1 and again in more detail in Ch. 4. A material with a negative derivative $dn/d\omega$ is anomalously dispersive and exhibits fast light pulse propagation, whereas a positive $dn/d\omega$ is called normal dispersion and leads to slow light pulse propagation [4]. The relationship between $v_g$ and $dn/d\omega$ is shown graphically in Fig. 1.2.

Although optical pulse propagation in dispersive media is in general quite complex, the group velocity approximation can often be used to describe pulse behavior in such media. By tailoring the dispersion in optical media, researchers have recently made pulses propagate very fast [7], very slow [8], and have even stopped light pulses altogether [9, 10]. These systems, in which the group velocity $v_g$ takes on extreme values, have sparked renewed discussion of the velocity of information encoded on optical pulses that travel much slower or faster than $c$. 
FIGURE 1.2: Group velocity as a function of dispersion. In this plot, for which \( n(\omega_0) = 1 \), the group velocity is shown as a function of \( \omega dn/d\omega \). For positive \( dn/d\omega \) (normal dispersion), \( 0 < v_g < c \). For negative \( dn/d\omega \) (anomalous dispersion), either \( v_g > c \) or \( v_g < 0 \). Negative group velocities are considered “fast” for reasons discussed in Sec. 2.2.

1.2 The information velocity debate

It has been known for more than a century that the group velocity of pulses of light can exceed \( c \). It was also long-believed that \( v_g \) was the velocity of information. Before Einstein formulated his special theory of relativity, faster-than-\( c \) information was not a concern. However, when he presented his theory in the early 20\textsuperscript{th} century, the community erupted in a flurry of concern over the apparent contradiction [2]; special relativity said “nothing” could go faster than \( c \), yet pulses of light can exceed \( c \) in anomalously dispersive media!

Special relativity states that normal matter and energy cannot travel faster than \( c \). More generally, it describes the observed timing of events. If two events

\[^1\text{It is possible that the energy velocity of a propagating electromagnetic field exceed } c \text{ if it exchanges energy with the medium in such a way that there is no net transmission of energy faster than } c \text{ [11].} \]
at different locations occur at the same time for one observer, they may occur at different times for another observer. If these two events are related—one is a cause and the other its effect—then the observed time and distance between them can be used to calculate the velocity of the information that travels between them. If the two events happen so close together in time that one observer concludes the information traveled faster than \( c \), relativity tells us that another observer will see the effect happen before the cause. As a result, it is now believed that no information can travel faster than \( c \); this requirement is known as Einstein, or relativistic, causality [11, 12].

It is not clear that the researchers of the early 20th century made the complete connection between the velocity of information and causality, but they certainly became concerned about group velocities faster than \( c \). Leading the research in this area were A. Sommerfeld and L. Brillouin. Their work, described in more detail in Sec. 3.2.1 continues to be cited today. They explored theoretically the propagation of square pulses through anomalously dispersive media and found that their pulses were badly distorted. They showed that the front of a pulse—the first point with a non-zero electric field—travels at \( v_f = c \). This convinced them (and many others for years to come) that \( v_i \) did not exceed \( c \). However, it did not address the question of how fast information really does travel.

To address the more general question of the true velocity of information, Sommerfeld and Brillouin defined the *signal velocity* \( v_s \) as the velocity of the main body of the pulse. Specifically, they defined \( v_s \) to be the velocity of the point where the pulse first reaches half its maximum height.\(^2\) They showed that when \( 0 < v_g < c \)

\(^2\)Actually, they defined \( v_s \) several different ways, each way having its own advantages and disadvantages. The definition presented here was more favored by Brillouin than by Sommerfeld. See Sec. 3.2.1 for more detail.
the pulse propagated at $v_g$ and that $v_s \approx v_g$. However, when $v_g$ was faster than $c$, the pulse did not propagate at $v_g$, but at $v_s$, which was slower than $c$.

For many decades, the work of Sommerfeld and Brillouin was accepted as a demonstration not just that information travels no faster than $c$ for fast $v_g$, but also that $v_g$ has no meaning when faster than $c$ [4]. This belief persisted until 1970, when Garrett and McCumber showed theoretically that smooth pulses (recall that Sommerfeld and Brillouin dealt with square pulses) can travel at fast group velocities with little distortion [13]. Their work was confirmed experimentally by Chu and Wong in 1982 [14] and again by Ségard and Macke in 1985 [15], re-opening the debate.

In the intervening years, few researchers have claimed that information can propagate faster than $c$. However, explanations for why information does not travel faster than $c$ are rare; predictions of how fast information does travel are virtually non-existent. One notable exception is a proposal by Chiao and collaborators which states that information encoded on electromagnetic waves always\textsuperscript{3} travels with velocity $c$ [11, 12]. Their proposal suggests that the pulse front—the first point where the electromagnetic field is non-zero—contains information.\textsuperscript{4} However, Chiao’s proposal goes farther, saying that all information is encoded in points of non-analyticity. These non-analytic points (of which the front is only one example) are special because the behavior of a waveform before such a point cannot be used to predict the behavior after the non-analytic point. Also, these non-analytic points propagate with the same velocity as the pulse front $v_f$, which is clear in linear media where any waveform can be thought of as a superposition of waveforms,

\textsuperscript{3}They really do mean “always”, and they really do mean $c$, even when the background refractive index is greater than (or less than) one.

\textsuperscript{4}Sommerfeld also believed that the front contained information [2].
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each component waveform having a single front, before which it is zero and after which it is analytic. It has been shown by many researchers (including Brillouin and Sommerfeld) that pulse fronts propagate at $v_f = c$. As a result, according to Chiao and coworkers, $v_i = v_f = c$.

Chiao’s proposal applies to all information on optical pulses, not just “fast” pulses. Specifically, they suggest that the information velocity $v_i$ is also equal to $c$ for slow pulses. This is in direct contradiction to the common wisdom concerning slow pulses. It is widely believed that $v_i = v_s \approx v_g$ for slow light pulses. This widespread belief is probably the reason that information on slow pulses has received such little attention from the research community.

Currently, there is no consensus within the research community about the velocity of information. Both the theoretical and experimental bodies of work are riddled with gaps and contradictions. Of all the current research, Chiao’s proposal is perhaps the most promising, but it is by no means accepted by all. More work must yet be done to address these issues.

1.3 Measuring the information velocity in fast- and slow-light media

To resolve the questions described above, it is necessary to measure $v_i$ experimentally. This, too, has been a controversial topic: some researchers claim faster-than-$c$ information transfer [16], while others report $v_i \approx c$ for fast light but $v_i \approx v_g$ for slow light [17]. Unfortunately, the existing techniques for measuring the velocity of information are unsatisfactory, largely because there is no clear link connecting the measured quantities and any formal definition of information. I have developed a
new technique for measuring the information velocity based on information-theoretic concepts and applied this technique to fast- and slow-light media.

### 1.3.1 Creating fast- and slow-light media

In order to measure the information velocity on fast- and slow-light optical pulses, I create two media; one with a very fast group velocity and one with a very slow group velocity. Both media, described in detail in Ch. 4, are based on the process of stimulated Raman scattering in a $^{39}$K vapor. By creating Raman gain peaks—frequency regions in which light is amplified—it is possible to tailor the dispersion of media to create very fast or slow group velocities.

Recall that fast or slow light can occur in spectral regions of anomalous and normal dispersion, and that dispersion is variation in the refractive index $n$. Because $n$ is intimately related to the gain or absorption properties of a medium by the Kramers-Kronig relations [4], manipulation of a medium’s gain properties can be used to create a fast- or slow-light medium [11,18–20]. Creating a single gain line results in normal dispersion and a slow-light spectral region, as shown in Fig. 1.3 a–c. Figure 1.3 d–f shows a gain doublet, or pair of gain lines, between which there is a fast-light spectral region.

Creating gain lines using the stimulated Raman process in $^{39}$K has a number of advantages. One is that the gain features in $^{39}$K are spectrally narrow, allowing us to create large dispersion. Another advantage is that the strength and frequency of Raman gain can be easily controlled. This is because the Raman process, discussed more fully in Sec. 4.1.1, is a nonlinear interaction involving not only the atoms and the amplified (or “probe”) field, but also an additional “pumping” field, as shown in Fig. 1.4. The amplification occurs when a probe photon of frequency
Figure 1.3: Single and double gain lines. For the single gain line (a–c), the central region is normally dispersive \((dn/d\omega > 0)\) and has a slow group velocity. In contrast, the central region of the gain doublet (d–f) is anomalously dispersive \((dn/d\omega < 0)\) and has a fast group velocity.
Figure 1.4: The atomic transition diagram for Raman gain. The atoms are first prepared in state $|a\rangle$. This preparation is discussed in Sec. 4.1.3.

$\omega_p$ stimulates this transition, creating a new $\omega_p$ photon and annihilating pump (or drive) photon of frequency $\omega_d$. In this process, the atom changes from state $|a\rangle$ to state $|b\rangle$. Conservation of energy requires that $E_a - E_b = h(\omega_p - \omega_d)$, so changing the pumping frequency $\omega_d$ changes the gain frequency $\omega_p$. Also, increasing the intensity of the pumping field increases the gain simply because it increases the likelihood that a pump photon will be physically present when the probe photon passes near the atom. Therefore, by changing the intensity and frequency of the applied pumping field, one can control the strength and frequency of the Raman amplification.

I use this Raman process to create both fast- and slow-light media. In the slow-light case, a single pump beam is used to create a single gain line in a $^{39}$K vapor cell, as shown in Fig. 1.5a. When 265-ns-long pulses are transmitted through this system, they are delayed by $t_{del} = 67.5$ ns compared to identical pulses traveling through vacuum. As shown in Fig. 1.5b, this is a delay of approximately 25% of their width. Given the cell length $L_s = 20$ cm, this delay corresponds to a group velocity of $v_g \approx 0.01c$. Even though $n(\omega) \approx 1$, slow group velocities can be produced.
by manipulating the dispersion.

The fast-light system is similar to the slow-light system, except that it uses two vapor cells and two pumping beams, as shown in Fig. 1.6a. It uses two cells to avoid a competing nonlinear effect described in Appendix A. The frequency of the probe pulses is tuned to the region between the two generated gain lines. Because the Raman process is linear in the probe field, these physically separate regions can be treated as a single composite region. Through the fast light medium, 263-ns-long pulses are advanced by $t_{\text{adv}} = 27.4$ ns, which is 10.4% of their width and corresponds to a group velocity of $v_g \approx -0.05c$, as shown in Fig. 1.6b. Such large advancement and little distortion are difficult to achieve because of the many requirements; large gain at two frequencies while avoiding nonlinear optical effects, low pulse power to avoid saturation, and nearly linear variation in the $n$. An advancement of 10% of the pulse width with so little distortion is truly remarkable and leaves no room for doubt about whether faster-than-$c$ group velocities are possible.

These fast- and slow-light media make it possible to test ideas about the information velocity when $v_g$ is very fast and very slow. They also provide large advancement and delay (respectively) relative to their vacuum counterparts. This allows me to clearly observe which aspects of the pulse shapes travel at $v_g$, and which travel at other velocities, such as $c$. This ability to distinguish the velocities of different pulse features is essential for measuring the information velocity.

### 1.3.2 Technique for measuring $v_i$

To measure the information velocity in these fast- and slow-light media, I have created a technique based on information-theoretic principles. The technique uses an alphabet of multiple distinguishable symbol shapes and a statistical measure of
Figure 1.5: Slow-light setup and pulses. (a) The experimental setup used to generate slow-light optical pulses. (b) Gaussian pulses after propagating through the slow-light setup (dashed line) and through vacuum (solid line). The pulses shown here are averages of 50 individual pulses and have been smoothed.
Figure 1.6: Fast-light setup and pulses. (a) The experimental setup used to generate fast-light optical pulses. (b) Gaussian pulses after propagating through the fast-light setup (dashed line) and through vacuum (solid line). The pulses shown here are averages of 50 individual pulses and have been smoothed.
detected information to quantify the information transmitted through a communications system. Any communications system has three major components: the sender, who chooses symbols and then sends them; the medium, through which the symbols propagate; and the receiver, who detects and identifies the symbols. I create such a system and use it to measure the velocity of information.

The first step in creating such a communications system is to choose an alphabet of symbols. The alphabet choice is largely arbitrary, but it can affect the experimental uncertainty and complexity of the interpretation. For example, the alphabet can consist of any integer number of distinct—or more to the point, distinguishable—pulses. However, interpretation and analysis are greatly simplified by using the minimum number of symbols: two. Therefore, I use an alphabet consisting of two symbols, each of which is a combination of Gaussian pulses. Both symbols have identical Gaussian leading edges, but near their peaks, the two symbols diverge; one symbol drops to zero intensity, and the other doubles in intensity and continues to complete its new Gaussian shape. The two ideal symbol shapes are shown in Fig. 1.7. These symbols are appealing both because they are similar to the Gaussian pulses previously transmitted through the fast- and slow-light media, and also because they contain clear discontinuities that Chiao suggests contain the information [11, 12].

With a symbol alphabet chosen and the media prepared, the remaining step is to devise a technique for identifying the received symbols. Successful communication of information requires that the receiver correctly identify the incoming symbols. The identification need not be perfect, but it must do better than would

\[5\] Within the communications community, it is common to use the term “detection” to refer to the combined process of measuring the incoming waveform and identifying the symbol. In the physics community, it is more common to use “detection” to describe only the measurement step. Throughout this thesis, I will largely follow the physics convention.
random guessing; if it is no better than random guessing, no information is being communicated.

1.3.3 The role of noise and distortion in measuring $v_i$

If we lived in a world without noise or distortion, it would be easy to identify the symbols; each symbol would be transmitted perfectly every time, and one could simply wait until the first moment when the two symbols differ. If we could know the moment the information is sent ($t_0$) and the moment it arrives ($t_1$), it would be possible to measure the information velocity through a medium of length $L$ using the simple formula

$$v_i \equiv \frac{L}{t_1 - t_0}.$$  

(1.3)
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However, this ideal conceptualization of the problem ignores the very real phenomenon of noise and distortion of the symbol shapes. In this section, I will address these issues, present a modified conceptualization that incorporates these phenomena, and describe a technique for minimizing their impact on the measurement of $v_t$.

The ideal picture described above is the scenario considered by Einstein and his contemporaries, as well as by most modern researchers. The conventional way to think about information propagation is to imagine a moment, known with absolute certainty, when information is sent, and a similarly well-known moment when the information is received. This concept appears throughout the popular and technical literature.

This idealization is not incorrect; it is simply incomplete as a physically realizable theory. In reality, noise and distortion, as described below, are unavoidable aspects of information transmission on optical pulses. Together, they make it difficult to know precisely when the information is sent or received. Noise is the random fluctuation of the detected quantity. There are many sources of noise, and all parts of the communications system, not just the medium, contribute to the noise. Distortion is a repeatable change in the shape of the symbols due to the properties of the system. Again, all components of the communications system can contribute to the distortion. An example of distortion is the smoothing of the optical waveform due to the finite response time of the detector.

If there were only distortion but no noise, one could still measure $t_0$ and $t_1$ perfectly. For example, imagine the symbol shapes in Fig. 1.7 as they might be detected by a detector with a finite response time. Instead of the discontinuities at $v_t$.

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6 The fluctuations need not be truly random to be considered noise, just practically unpredictable.
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time $t = 0$ ns being infinitely sharp, the detector would smooth them so that the symbols separated slowly. Immediately after time $t = 0$ ns, the symbols will still differ, but they will differ by less. For example, this smoothing might cause the ideal symbol intensities to be separate by only 0.01 rather than 2 at time $t = +1$ ns; nonetheless, the symbols will be distinguishable if one observes with high enough resolution. Distortion alone doesn’t prevent or even delay identification, it only requires that one look closer.

The real trouble comes when noise is also present, as it always is in any real measurement. With noise, one can no longer look as closely as required in order to overcome the distortion. For example, consider the case described above, when distortion causes there to be a difference of only 0.01 between the intensities at $t = +1$ ns. If there is also 0.1 unit of noise, then the symbols cannot be distinguished with absolute certainty at $t = +1$ ns with a single observation.

In order to address these issues of noise and distortion, I have developed a modified version of the ideal picture described above. My model is based on the ideal case; it assumes that there is a unique moment ($t_0$) when information is first sent and another moment ($t_1$) when information is first received. However, the information cannot be detected until some time $t_c$ after it has been received. That is, a symbol arrives at the detector, but due to noise and distortion, it is only identified at some later time. Even then, the identification is not certain, but only probable. This delay can be reduced by performing the identification earlier, but only at the cost of less certain identification. However, the receiver can never identify a symbol before $t_1$ because the information is not yet available at the detector. As a result, $t_c \geq t_1$. I call the delay $t_c - t_1 = \Delta t$ the detection latency.

In any measurement of the information velocity, it is important to minimize and
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characterize the detection latency. By ignoring the detection latency, it is easy to conclude that information propagates much faster (or slower) than it really does. One of the most important contributions of this work is the analysis of the detection latency and its role in information transmission. A more thorough discussion of the detection latency is presented in Secs. 5.2.1 and 5.2.2.

I have developed a technique for minimizing the impact of the detection latency, but it cannot be eliminated altogether. The technique I have developed, described in more detail in Ch. 5, identifies the incoming pulses using a matched-filter comparison. This technique compares each incoming pulse to two “reference pulses,” each of which represents the ideal shapes of one of the two received symbols. The technique quantifies the similarity of the incoming pulse to the two references. By measuring the similarity between the incoming pulse and each reference pulse, the incoming pulse can be identified.

Using this identification technique, it is possible to form a statistical measure of detected information. The information is quantified by the bit error rate (BER). The BER, just as it sounds, is the fraction of incoming bits that are mis-identified. If the chosen alphabet consists of two symbols, then each equi-probable symbol carries a single bit of information and the BER is just the fraction of symbols that are misidentified. For reasonable identification techniques, the worst possible BER is 1/2; for a BER of 1/2, one might as well just guess which symbol was sent. Therefore, a BER of 1/2 implies that no information is being successfully communicated. Any BER smaller than 1/2 indicates communicated information. Even for a BER of 0.499, information is being received; a large BER such as this simply represents a low level of confidence in an individual symbol’s identification. However, the observation can always be repeated to improve this confidence. Therefore, each incoming
symbol must be carrying a little more information.

While this identification technique helps deal with the detection latency, one must also compensate for large delays in the sending and detection stages. These delays come from the electronic processing and optical propagation of the symbols outside the medium. Because only the velocity inside the medium is of interest, these delays must be eliminated. This problem can be addressed by also sending the symbol pulses through a length of vacuum equal to the length of the medium. Because both cases share these external delays, they can be compared to isolate the effects of the medium. As discussed in Sec. 5.2.1, Eq. 1.3 can be reformulated to include this vacuum data.

The true power of this identification technique and BER measurement is that it can be applied as a function of time for incoming pulses, allowing the detected information to be quantified as a function of time. One can therefore use the BER measurement and Eq. 1.3 (modified to account for the detection latency and external delays) to find the information velocity for any medium.

1.4 Information velocity experiments

By combining the techniques described in the previous sections, I am able to measure the information velocities in these fast- and slow-light media. In both cases, I generate the media as described in Sec. 1.3.1 and measure the information velocity according to the technique described in Secs. 1.3.2 and 1.3.3. These experiments represent the culmination of this thesis, and are described in detail in Secs. 6.2 and 6.3.

Figure 1.8a shows the symbol pulses after propagating through both the fast-light medium and vacuum. Although the leading edges of the fast-light pulses
emerge earlier, the two fast-light symbols (dashed lines) can be distinguished no earlier than their vacuum counterparts (solid lines). This is shown more clearly in Fig. 1.8b, which is simply a zoomed-in version of (a). These results suggest that information encoded on the pulses cannot travel faster than $c$ in this medium even though smooth pulse shapes can.

The results of a similar measurement in the slow-light medium are shown in Fig. 1.8c, with a zoom shown in Fig. 1.8d. In this case, the pulses through the slow-light medium emerge later. However, like the fast-light case, the point where the symbols can be distinguished occurs at nearly the same time as in vacuum;
although the pulses propagate slowly, information encoded on them still propagates at speeds similar to vacuum speeds!

To better quantify the arrival time of the information, I perform the BER analysis described briefly above. The bit error rates for the fast- and slow-light experiments are shown in Fig. 1.9. In all cases, the BER is near 1/2 for early times when the symbols are indistinguishable. At later times, as the symbols begin to separate, the BER drops. Based on this data and a mathematical model of the media, it is possible to determine the information velocity in these fast- and slow-light media. I find that $v_{i,fast} = 0.4(0.7 - 0.2)c$ and that $v_{i,slow} = 0.6c$. In the slow-light case, the error bars are slightly more complicated; they include the range from $0.2c$ upward through $\pm\infty$ to $-0.5c$. These results are consistent with Chiao’s hypothesis that $v_i = c$ and are clearly inconsistent with the idea that $v_i = v_g$. 

**Figure 1.9:** BER curves for the fast- (a) and slow-light (b) pulses.
1.5 Summary

The experiment and experimental results described briefly in this introduction represent the first detailed measurement of the velocity of information on optical pulses. I have applied this measurement technique to pulses with extremely fast and slow group velocities. This work both helps address a debate that has been raging for a century, and also paves the way for future research into the velocity of transmitted information. For example, the techniques designed herein may be used to help characterize and optimize the information velocity in communications and computation devices.

The body of this thesis describes in much more detail the techniques and results described in this introduction. In Ch. 2, I discuss the fundamentals of fast and slow light, including negative group velocities, other techniques for generating fast and slow light, and other research in the field. Chapter 4 describes the preparation of the fast- and slow-light media used in my experiments. In Ch. 5, I describe the information velocity measurement technique and the detection latency. Finally, in Chs. 6 and 7, I present and discuss the results of my experiments and conclude the thesis.

1.6 Scientific acknowledgments and collaborative history

The purpose of this thesis is to present my scientific accomplishments. Therefore, I often use such phrases as “I measure” and “my work”. While such statements are completely correct, they are incomplete. I performed this research in collaboration with my thesis advisor, Daniel J. Gauthier, and with Mark A. Neifeld.
This project began when Dan and I heard about the experiments of Wang et al. [7], which used bichromatic Raman gain to generate fast light. We quickly realized that the size of the fast-light effect is directly related to the strength of the created Raman gain lines. Because Dan had spent years studying large Raman gain for other experiments [21–25] (with me and other students), we realized that we were uniquely equipped to extend Wang’s results.

After considerable refinement of Wang’s (originally, Steinberg and Chiao’s [26]) technique and the usual side-projects (see Appendix A and Ref. [27]), we were able to achieve very large pulse advancement, as described above. However, any researcher in this field must be familiar with questions of relativistic causality (see Sec. 3.1). In our study of this issue, we came to realize that the broader topic of the information velocity was (much to our surprise) poorly understood.

Dan and I felt strongly that if we were to attempt to scientifically study the velocity of information, we should at least begin with information and communications theory. Therefore, we began searching for a researcher who had the familiarity with these fields that we lacked, yet was familiar enough with the realities of experimental pulse propagation that we could effectively communicate. Our search led us to Mark A. Neifeld, from the University of Arizona.

The final product, as presented in this thesis, is very much the result of collaboration between all three of us. I performed all experiments and theoretical calculations herein, and contributed significantly to the theoretical framework that we developed. Dan, as the clear expert in experimental optical physics—and specifically, Raman gain—was fundamentally involved in the design and analysis of the fast- and slow-light systems, as described in Chs. 2–4. Mark taught us nearly everything we know about information and communications theory, and is therefore
largely responsible for the design of the information detection system described in Sec. 5.1. This research is funded by the U.S. National Science Foundation, grant #PHY-0139991.
Chapter 2

Fundamentals of fast and slow light

Simply put, fast and slow light pulses are pulses that travel faster and slower (respectively) than pulses propagating through vacuum. This phenomenon has been known, or at least predicted, for more than a century. It is completely consistent with Maxwell’s equations. But still, fast and slow light remain poorly understood by many, if not most, physicists and engineers. In this chapter, I will describe the physical and mathematical origin of fast and slow light, and various techniques for creating fast and slow light pulses.

2.1 Basic theory of fast and slow light

Throughout this document, the terms “fast” and “slow” are used to refer to the group velocity \( v_g \); a fast pulse is one with a group velocity faster than \( c \), a fast medium is one in which fast pulses propagate, etc. It is important to be clear that no other quantity, unless explicitly stated, is implied to be “fast” or “slow” when a pulse is so described. A lack of clarity has long plagued the word “superluminal” as it applies to this field; traditionally, “superluminal pulses” and “fast pulses” are synonymous, but many long and painful arguments have ended with “oh, you mean
the group velocity? Never mind, then.” To many people, the word “superluminal” appears to have relativistic connotations that are difficult to avoid. For this reason, I now avoid using the word “superluminal” in favor of “fast.”

It is worth mentioning that another phenomenon, called superlumino\_ous propagation is often confused with fast, or superluminal, pulse propagation. Superluminous propagation is a nonlinear phenomenon wherein a saturable gain medium amplifies the front end of a pulse and absorbs the back end of the pulse, causing the overall pulse shape to travel faster than $c$ [28]. This is an interesting phenomenon, but is distinct from the processes discussed here; for all of the processes discussed in this thesis, with the exception of the phenomena described in Appendix A, the response of the medium to the probe (or information-carrying) pulse is always in the linear regime.

### 2.1.1 The envelope equation

It is possible, making only a few reasonable approximations, to derive a general expression for the shape of an optical pulse after propagating through an arbitrary linear medium. The medium is characterized by the wavenumber $k(\omega)$, which is a function of frequency $\omega$. The pulse is described by its time-varying electric field at position $z = 0$. Usually, position $z = 0$ is the input of the medium. Using these, we can derive an expression for the temporal variation of the pulse at an arbitrary depth inside the medium.

The electric field associated with a quasi-monochromatic optical pulse can be represented as a slowly modulated monochromatic wave,

$$E(z, t) = A(z, t) e^{-i\omega t} + c.c.,$$

(2.1)
where $\omega_0$ is the central optical frequency of the pulse. The function $A(z,t)$ is called the “field envelope” or “pulse envelope” because it describes the general shape of the modulation without regard for the carrier frequency $\omega_0$. The pulse intensity $I(z,t)$ is the quantity that is usually measured experimentally, and is related to $A(z,t)$ by $I(z,t) = |A(z,t)|^2$.

In terms of the envelope, the field’s Fourier transform $E(z,\omega)$ is then given by

\[
E(z,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(z,t) e^{i\omega t} dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( A(z,t) e^{i(\omega-\omega_0)t} + A^*(z,t) e^{i(\omega+\omega_0)t} \right) dt. \tag{2.2}
\]

This integral can be performed trivially if we recall two common tools. The first is Parseval’s relation [29]

\[
\int_{-\infty}^{\infty} F(\omega) G^*(\omega) d\omega = \int_{-\infty}^{\infty} f(t) g^*(t) dt, \tag{2.3}
\]

where $F(\omega)$ and $G(\omega)$ are the Fourier transforms of $f(t)$ and $g(t)$. The second tool is the Fourier representation of the Dirac delta function [29]

\[
\delta(t - x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t-x)} d\omega. \tag{2.4}
\]

Using Eqs. 2.3 and 2.4 with the intermediate frequency variable $\omega'$, Eq. 2.2 becomes

\[
E(z,\omega) = \int_{-\infty}^{\infty} [A(z,\omega') \delta^*(\omega' - (\omega - \omega_0)) + A^*(z,\omega') \delta^*(\omega' - (\omega + \omega_0))] d\omega' = A(z,\omega - \omega_0) + A^*(z,\omega + \omega_0). \tag{2.5}
\]

If we neglect the backward-traveling solution and the pulse modulation is slow
compared to the optical frequency $\omega_0$, we can approximate $E(z,\omega)$ as

$$E(z,\omega) \approx A(z,\omega - \omega_0). \quad (2.6)$$

We see that the Fourier transform of the envelope has approximately the same functional shape as the Fourier transform of the field itself, except that the transform of $A(z,t)$ is centered about zero frequency, whereas the transform of $E(z,t)$ is centered about the carrier $\omega_0$.

If we now require that the field in Eq. 2.6 solve the wave equation [30]

$$\frac{\partial^2 E(z,\omega)}{\partial z^2} + k^2(\omega)E(z,\omega) = 0, \quad (2.7)$$

we find that the field at an arbitrary depth $z$ must be of the form

$$A(z,\omega - \omega_0) = A(0,\omega - \omega_0) e^{i k(\omega) z}. \quad (2.8)$$

Physically, this means that each frequency component of the pulse acquires a phase shift and attenuation or gain according to the properties of the medium. These properties of the medium are represented mathematically by the wavenumber $k(\omega)$.

We can now inverse transform Eq. 2.8 to find the field envelope as a function of time at an arbitrary depth $z$ in the medium.

$$A(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(0,\omega - \omega_0) e^{i(k(\omega)z - \omega t)} d\omega \quad (2.9)$$

This simple result is actually quite powerful, and will be used throughout this thesis both analytically and numerically.
2.1.2 The group velocity

In general, Eq. 2.9 is quite difficult to compute in closed form, especially because the mathematical forms of both $A(0, \omega - \omega_0)$ and $k(\omega)$ can be quite complicated. However, a number of basic approximations can be made by truncating the Taylor expansion of $k(\omega)$ about $\omega_0$.

$$k(\omega) = k_0 + k_1(\omega - \omega_0) + k_2 \frac{1}{2} (\omega - \omega_0)^2 + \ldots$$ (2.10)

Here, we use $k_j \equiv d^j k(\omega)/d\omega^j$. In general, $k(\omega)$ is a complex quantity. It is conventional to distinguish between the real and imaginary parts. The real parts are called the “dispersive” terms; they affect pulse propagation only by introducing phase shifts to the various frequency components. The imaginary parts are called “absorptive” terms; they result in absorption or gain.

The most common approximation is $k(\omega) \approx k_0 + (\omega - \omega_0)k_1$. This approximation describes a linearly dispersive medium—$k(\omega)$ depends on $\omega$ linearly and the linear coefficient $k_1$ is real. One example of such a medium is vacuum, where $k(\omega) = \omega/c$.

In this approximation, the integral in Eq. 2.9 can be performed exactly by adding $(\omega_0 t - \omega_0 t)$ to the exponent and employing Eqs. 2.3 and 2.4 to find

$$A_1(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(0, \omega - \omega_0) e^{i[[k_0 + k_1(\omega - \omega_0)]z - \omega_0 t + (\omega_0 t - \omega_0 t)]} d\omega$$

$$= e^{i(k_0 z - \omega_0 t)} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(0, \omega - \omega_0) e^{-i(\omega - \omega_0)(t - k_1 z)} d\omega$$

$$= e^{i(k_0 z - \omega_0 t)} \int_{-\infty}^{\infty} A(0, t') \delta(t' - (t - k_1 z)) dt'$$

$$= e^{i(k_0 z - \omega_0 t)} A(0, t - k_1 z), \quad (2.11)$$

which includes a constant attenuation (or gain) and phase shift from the leading
complex-valued $k_0$ exponential. The subscript “1” indicates that this has been computed using first Taylor expansion of $k(\omega)$ truncated to first order. Aside from the attenuation and phase shift, the pulse is transmitted without distortion at the velocity $k_1^{-1}$. If we recall that $k(\omega) = \omega n(\omega)/c$, then we see that

$$k_1 = \left[ \frac{d}{d\omega} \frac{\omega n(\omega)}{c} \right]_{\omega=\omega_0} = \frac{1}{c} \left( n(\omega_0) + \omega_0 \frac{dn}{d\omega} \bigg|_{\omega=\omega_0} \right) = \frac{1}{v_g},$$

(2.12)

where $v_g$ was defined in Eq. 1.2.

We see that the concept of a group velocity—the velocity at which a pulse propagates undistorted—emerges directly from the envelope equation (Eq. 2.9) in the approximation that $k(\omega) \approx k_0 + (\omega - \omega_0)k_2$. Note that this derivation does not depend on the shape of the pulse $A(0, t)$, but is exact for any pulse shape. However, that does not imply that it is a good approximation for any pulse shape. In the next sections, I will discuss conditions under which this approximation and others hold, and the affects of higher-order terms in the expansion of $k(\omega)$.

### 2.1.3 Group velocity dispersion and quadratic gain

In the previous section, we kept only the linear terms in the expansion of $k(\omega)$. However, the quadratic terms can also be of considerable importance. It is convenient to separate the quadratic component into its real and imaginary parts

$$k_2 = k'_2 + ik''_2,$$

(2.13)

where $k'_2$ is the real part and corresponds to group velocity dispersion (GVD), and $k''_2$ is the imaginary part and corresponds to a quadratic variation in the gain or absorption of the medium. With this approximation, the envelope equation can no
longer be solved analytically for an arbitrary pulse shape, but it can be solved for certain pulse shapes, including a Gaussian pulse

\[ A_g(0, t) = A_0 e^{-t^2/2t_f^2}, \]  
\[ A_g(0, \omega - \omega_0) = A_0 t_f e^{-\frac{1}{2}t_f^2(\omega - \omega_0)^2}, \]

where \( A_0 \) is the field magnitude at the pulse peak and \( t_f \) is the \( 1/e \) intensity temporal half-width of the pulse. The subscript "g" indicates that these quantities are for a Gaussian pulse. Because both the approximate \( k(\omega) \) and \( A_g(0, \omega - \omega_0) \) are exponentials of quadratic polynomials, the integral in Eq. 2.9 can be performed analytically by completing the square in the exponent. The result of the integration with the second-order \( k(\omega) \) approximation is

\[ A_{g2}(z, t) = A_0 \left( 1 - \frac{ik_2 z}{t_f^2} \right)^{-1/2} e^{i(k_0 z - \omega_0 t)} \exp \left[ -\frac{1}{2} \left( \frac{t - k_1 z}{t_f^2 - ik_2 z} \right)^2 \right]. \]  

Here, the first quantity in parentheses controls the modification in pulse height from \( k_2 \). The pulse shape itself is determined by the final exponential. Note that the pulse again propagates at the group velocity \( k_1^{-1} \), but there is now a modification of the pulse width. The new width is determined by the denominator of the exponent. Imaginary values of \( k_2 \) correspond to quadratic gain and can lead to either pulse compression or expansion. Real values of \( k_2 \) produce only pulse expansion and also introduce pulse chirp or variation of the instantaneous frequency as a function of time. The pulse grows or shrinks in height by the same factor with which it is compressed or expanded. Note the when \( t_f^2 = ik_2 z \), the pulse acquires an infinitely short but large envelope. We can interpret this as a reasonable limit on the usefulness of this approximation. In practice, higher-order terms will become relevant before
this limit is reached.

### 2.1.4 Usefulness of expansions of $k(\omega)$

It is important to remember that the usefulness of the approximations described above will depend on the application. In general, the accuracy of such approximations depends both on the medium and on the pulse. Also, some applications require much greater accuracy than others.

If, when discussing the propagation of a pulse through some medium, you are ever asked “is the pulse distorted?”, you can confidently answer “yes, of course!” Vacuum is the only physical medium where all terms higher than first order are zero. In all other media, a pulse will be distorted.\(^1\) The *important* question is whether the pulse is intolerably distorted. While this may seem like a fine point, it is nonetheless an important one.

For any given pulse and medium, the contribution from the $j$-th term in $k(\omega)$ can be estimated by considering the magnitude of the quantity

\[
L_j = \frac{(T_0)^j}{|k_j|},
\]

where $T_0$ is a measure of the width of the pulse. The quantity $L_j$ is the characteristic length scale over which the $j$-th term in $k(\omega)$ is significant. For example, if $L_4 = 1\, \text{m}$, then the fourth term in the $k(\omega)$ expansion will appreciably affect the pulse shape when it propagates for distances greater than approximately $1\, \text{m}$. These lengths can also be used to compare different orders to see which are of greater concern for a given application. For example, typical values of $L_2$ and $L_3$ for $1.31\, \mu\text{m}$ pulses

\(^1\)Typically, the effects of $k_2$ and higher terms are considered “distortion,” while the effects are of $k_0$ and (real) $k_1$ are not because the latter terms do not affect the pulse shape.
with $T_0 = 100$ ps in optical fibers are $L_2 = 10^4$ km and $L_3 = 10^7$ km [6]. Therefore, if one needs to transmit pulses around the world (approximately $10^4$ km) one might expect the $k_2$ term to produce significant distortion, but the $k_3$ term to have no significant effects.

Another important consideration when addressing the usefulness of an approximation is whether the relevant pulse feature is a rapid fluctuation in the pulse shape. An example of such a feature is the pulse front, the first moment when the field is non-zero. The same arguments used for the pulse itself hold for the propagation of the feature; the importance of each term in $k(\omega)$ depends upon the temporal width of the feature. In the extreme case of an infinitely sharp feature like the front, all of these approximations will fail.

Finally, it is not safe to assume that the Taylor series expansion of $k(\omega)$ converges for any non-zero frequency range. Careful attention must be paid to the specific system and the frequency range of interest.

### 2.2 Quantifying the “fastness” of light

In the previous sections, I described many general and specific properties of pulse propagation. That discussion focused on the mathematical derivation of physical phenomena, with little emphasis on the interpretation of those phenomena. In this section, I will discuss the interpretation of the group velocity and other ways to quantify the “group velocity effect.”
2.2.1 Spacetime diagrams for the group velocity

The “group velocity effect” applies well when \( k(\omega) \approx k_0 + (\omega - \omega_0)k_1 \), in which case, the pulse will travel at the group velocity \( v_g = k_1^{-1} \) and retains its shape. In that case, we can simply concern ourselves with the peak of the pulse as it propagates. It is then illuminating to create a spacetime diagram for the pulse propagation.

Figure 2.1 contains several spacetime diagrams for pulses with different group velocities. In each figure, the pulse propagates through a medium of length \( L \). The simplest spacetime diagram is for a pulse traveling through vacuum, as shown in Fig. 2.1a. Before the medium, the pulse propagates with speed \( c \), which is shown by the slope of the line. If the medium (shaded region) is also vacuum, then the pulse continues through the medium at speed \( c \). Figure 2.1b shows the same diagram for a pulse in a slow-light medium. The dashed line shows how the pulse would have propagated through vacuum. Again the slope of the line is equal to \( 1/v_g \), and the pulse group delay \( t_g \) (relative to a vacuum pulse) is shown on the right.

Figure 2.1c describes the motion of a pulse with a fast group velocity \( v_g > c \), as demonstrated by the smaller slope of the line inside the medium. In this case, the group delay \( t_g \) is negative. Figure 2.1d shows the propagation of a pulse with \( v_g = \infty \), depicted by a zero-slope line in the medium. This simply means that the peak exits the medium at the same moment it enters, and exists throughout the medium at that moment.

The final diagram, Fig. 2.1e, shows a pulse propagating with a negative group velocity. At early times, the peak is far in front of the medium. At some point before the peak enters the medium, peaks form at the exit face of the medium and propagate both forward away from the medium and backward into the medium. The peak propagating backward inside the medium eventually meets the incoming
Figure 2.1: Spacetime diagrams for pulses traveling through media with various group velocities: (a) $v_g = c$, (b) $0 < v_g < c$, (c) $c < v_g$, (d) $v_g = \infty$, and (e) $v_g < 0$. The solid line shows the spacetime trajectory of the pulse peak. Note that observation of a horizontal slice through the diagram corresponds to a snapshot, showing where the pulse is at that moment. In the case of $v_g < 0$, there can be multiple simultaneous peaks.
peak at the entrance face.

Note that throughout this progression from slow to fast, the group velocity went through infinity and became negative, but \(1/v_g = k_1\) and \(t_g\) varied continuously.

### 2.2.2 The physics of negative group velocities

The group velocity phenomenon that puzzles people most is the concept of a negative group velocity. When \(v_g < 0\), a pulse peak can be observed to travel backward in the medium and to exit the medium before it enters, as shown in the previous section. For a brief time after the peak leaves and before the peak enters, one can observe three peaks simultaneously; one before the medium, one after, and one inside. These phenomena can be easily explained in terms of the wavenumber \(k(\omega)\) and interference of different component frequencies.

Before delving too deeply into the mathematical description of this effect, let us first consider an analogy. Imagine three competitors in a triathlon; we’ll call them A, B, and C. In the first part of the triathlon, running, A is the fastest, B is a bit slower, and C is slower still. For the second stage, swimming, the order is reversed. For the final stage, biking, the order is the same as for running. At the start of the race, all of the competitors are in the same place, but they quickly begin to spread out as A takes the lead and C falls behind. However, as soon as they enter the water, the gaps begin to narrow because A is now slowest and C fastest. At some point as they are swimming, they are all neck-and-neck again, but only for a moment because C pulls ahead and A falls behind. Later still, they emerge from the water and begin biking. Again, A is fastest and C slowest, so as they approach the finish line, it is a photo-finish.

In many ways, our competitors are like individual frequency components of a
light pulse travelling through different media. By reversing the dispersion of the central medium, the competitors are in phase three times instead of the normal one time that would occur in a marathon. The obvious difference is that each of our competitors only exist at a single location at each moment, whereas our monochromatic frequency components exist for all positions and times. Nonetheless, the underlying physics is the same, as I will show in the next example.

Each monochromatic frequency component has some phase $\phi$. When all of the components have the same $\phi$, they constructively interfere, and the pulse has a peak. This can be represented mathematically as

$$\frac{d\phi}{d\omega} = \frac{d}{d\omega} \left( k(\omega) z - \omega t \right) = k_1 z - t = 0.$$  (2.18)

Requiring that $d\phi/d\omega = 0$ leads to a tidy demonstration that $\nu_g = k_1^{-1}$. However, we can also use $d\phi/d\omega$ as a measure of how in-phase the various frequency components are. Note that $d\phi/d\omega$ has units of time; it is the time lag between the peaks of two similar frequencies. This is exactly analogous to the time lag between our competitors A and B.

Figure 2.2a shows a pulse propagating through a medium with $\nu_g = -2c$ and Fig. 2.2b shows the corresponding $d\phi/d\omega$. The pulse is moving from left to right. This is an example of a situation where the pulse peak leaves the medium before it enters. As a result, there is a peak travelling backward (right to left) inside the medium. Before the first peak, similar frequencies are out of phase with negative phase differences ($d\phi/d\omega < 0$). Looking farther forward, the phase differences become small at the peak. Continuing forward, the frequency components start to become out of phase again, now with $d\phi/d\omega > 0$.

At the entrance to the medium, the phase difference begins to shrink again as
Figure 2.2: (a) A pulse propagating through a medium with $v_g < 0$ and (b) $d\phi/d\omega$ as a function of position for the same pulse. Both quantities are plotted as a function of position at a single moment in time. The central region is the fast-light medium with $v_g = -2c$, with vacuum before and after.
the previously diverging frequencies begin to reconverge. This is analogous to our triathletes who were diverging before they entered the water, but were reconverging as soon as they entered the water. An important difference between the triathletes and our frequency components is that the frequency components exist for all time and space. Therefore, Fig. 2.2b shows $d\phi/d\omega$ as a function of position for a single instant in time. To plot the time difference for our triathletes as a function of position, one must allow time to vary as they travel.

If Fig. 2.2b seems familiar to you, it should. It is identical to the spacetime diagram Fig. 2.1e. Recall that the $d\phi/d\omega$ at a given position is simply the time it will take for the pulse peak to arrive at that position, which is exactly the meaning of the spacetime diagram. Also, as seen from Eq. 2.18, the slope of the $d\phi/d\omega$ plot is equal to $k_1 = 1/v_g$.

As described in Sec. 1.1, an optical pulse is not a discrete object. Therefore, it can have multiple peaks simultaneously. Because of these two peaks, the emerging pulse is often considered a different pulse from the one that entered. However, both the emerging and entering peaks are composed of the same underlying frequency components and so both have an equally strong claim on the label of “original pulse.”

### 2.2.3 Group velocity vs. advancement

While negative group velocities are quite physically meaningful and mathematically consistent, they are a bit cumbersome to discuss. For example, why is a pulse with $v_g = -0.01c$ faster than one with $v_g = 10^{10}c$? The short answer is that the former comes out earlier, as shown in Fig. 2.1. To quantify this exit time, let us define the pulse advancement $t_{adv}$ as the amount of time by which a pulse is advanced relative
to an identical pulse traveling through vacuum. This is conceptually identical to
the group delay $t_g$ or the pulse delay $t_{del}$ but differs in sign. All of these may be
used in different contexts, but they have the same meaning. These quantities are
related by

$$-t_{adv} = t_{del} = t_g = \frac{L}{v_g} - \frac{L}{c} = L \left( k_1 - \frac{1}{c} \right)$$  \hspace{1cm} (2.19)$$

There are many reasons why the pulse advancement or delay is often a more
useful measure of the “fastness” of a pulse. As shown in Fig. 2.1, the group delay
varies smoothly as pulses go from very slow (small positive $v_g$) to very fast (small
negative $v_g$). This is shown more clearly if we consider $v_g$ and $t_g$ as functions of
$\omega \, dn/d\omega$, as shown in Fig. 2.3. The transition from slow to fast (and to negative
velocities) is smooth and intuitive in terms of the group delay.

Another reason that the advancement or delay is often more appealing than $v_g$
is that it is better linked to the physical limitations and practical usefulness of the
phenomenon. For example, imagine a 1-m-long medium constructed in the lab that
can propagate 10-ns-long pulses at a maximum group velocity of $v_g = 1000c$. This
corresponds to an advancement of approximately 3 ns. While that seems small, it
could be useful if it could be used to send pulses around the world at 1000c. Sending
a pulse 10,000 km would produce an advancement of 30 ms, which is quite useful
given that this how long it takes a pulse to travel at $c$! By using this system, one
could eliminate the propagation delay.\(^2\) However, one cannot simply extend this
laboratory system with $L = 1$ m to $L = 10^7$ m. For reasons that will be explained in
Sec. 2.3, the maximum $v_g$ is related to the medium length $L$ and the advancement
is not. Therefore, if the maximum advancement is 3 ns for $L = 1$ m, then it is also
3 ns for $L = 10^7$ m.

\(^2\)Let us ignore the question of information velocity for the moment.
Figure 2.3: (a) $v_g$ as a function of $\omega \, dn/d\omega$ and (b) $t_g$ as a function of $\omega \, dn/d\omega$. In both cases, the dashed line represents fast light and the solid line represents slow light. For these plots, $n_0 = 1$. 

While the maximum advancement is not related to the length of the medium, it is related to the pulse length. This will be discussed in more detail in Sec. 2.3, but the limitation comes from the relationship between the pulse spectral width and the spectral width of the region of anomalous dispersion. Using the example above, if the maximum advancement 3 ns for a 10-ns-long pulse, then it will be 0.3 ns for a 1-ns-long pulse. It is therefore convenient to define the relative advancement

$$A = \frac{t_{adv}}{t_w},$$

(2.20)

where $t_w$ is the pulse intensity full-width at half-maximum (FWHM). This quantity nicely captures the physical limitations of fast-light pulse propagation. Currently, researchers have been limited to $A < 0.5$ without severe pulse distortion.

2.3 Techniques for creating fast and slow light

As shown in the previous section, the creation of fast and slow light requires a medium with extreme values of $k_1$, the first order term in the expansion of the wavenumber $k(\omega)$. It is also usually desirable to minimize the effects of higher-order terms. These goals can be achieved in many different ways. In this section, I describe several techniques for generating fast and slow light. The first two techniques, a single gain or absorption line and a dual gain or absorption line, are the techniques that I use in the experiments described in Chs. 4 and 6. The other techniques are commonly used by other researchers and are presented here in the interest of completeness.

When possible, the results herein are derived in mathematical generality; for example, the treatment of a single line is equally applicable to a single gain line or
a single absorption line, depending only on the value of the absorption coefficient \( \alpha_0 \).

### 2.3.1 Gain or absorption

The simplest and best known technique for generating slow or fast light is to transmit pulses through a region with a single gain or absorption line. This is precisely the situation that concerned Sommerfeld and his contemporaries in the early 20\(^{th}\) century; they knew that the group velocity exceeds \( c \) in medium with an absorption resonance as described in this section.

A medium with a single gain or absorption line can be described by the complex index of refraction (which includes both dispersive and absorptive effects), which takes the form

\[
n(\omega) = n_\infty - \frac{c}{\omega} \frac{\alpha_0}{2} \frac{\omega - \omega_0 + i\gamma}{\omega - \omega_0 + i\gamma},
\]

where \( \omega_0 \) is the resonant frequency, \( \gamma \) is the half-width at half-maximum (HWHM) of the resonance, \( n_\infty \) is the background refractive index and \( \alpha_0 \) is the line-center intensity absorption coefficient \([13]\). If \( \alpha_0 > 0 \), this describes an absorption line, and if \( \alpha_0 < 0 \), it describes a gain line. The intensity of a monochromatic beam of frequency \( \omega_0 \) will be attenuated or amplified according to

\[
I(z) = I(0) e^{-\alpha_0 z}.
\]

From Eq. 2.21 and recalling that \( k(\omega) = n(\omega) \omega/c \), we find

\[
k(\omega) = \frac{\omega}{c} n_\infty - \frac{\alpha_0}{2} \frac{\gamma}{\omega - \omega_0 + i\gamma}.
\]
Expanding this $k(\omega)$ as described in Sec. 2.1.2, leads to the following coefficients:

\begin{align*}
    k_0 &= \left(\frac{\omega_0}{c} n_\infty + i \frac{\alpha_0}{2}\right), \quad (2.24) \\
    k_1 &= \frac{1}{c} \left( n_\infty - \frac{\alpha_0 c}{2\gamma} \right), \quad (2.25) \\
    k_2 &= -i \frac{\alpha_0}{\gamma^2}, \quad (2.26) \\
    k_3 &= \frac{3\alpha_0}{\gamma^3}, \quad (2.27)
\end{align*}

After the first coefficient $k_0$, the coefficients alternate between pure real and pure imaginary due to the symmetry of the resonance line. This means that the odd terms are purely dispersive and the even terms are purely absorptive.

Because a complex $k(\omega)$ is difficult to visualize and because the real and imaginary parts affect the light pulse in physically different ways, it is conventional to separate $k(\omega)$ into its dispersive and absorptive parts according to

\begin{align*}
    n(\omega) &= \Re \left[ \frac{c}{\omega} k(\omega) \right] ; \quad \alpha(\omega) = \Im [2k(\omega)]. \quad (2.28)
\end{align*}

Here, the real index of refraction $n(\omega)$ describes the purely dispersive properties of the medium and the absorption coefficient $\alpha(\omega)$ describes the purely absorptive properties. These quantities are plotted in Fig. 2.4 for a single gain line along with the $n(\omega)$ and $\alpha(\omega)$ values corresponding to the second-order expansion of $k(\omega)$.

The value of $k_1$ in Eq. 2.25 can be used to calculate the group velocity for a single gain or absorption line according to

\begin{align*}
    v_g = \frac{1}{k_1} = \frac{c}{n_\infty - \frac{\alpha_0 c}{2\gamma}}. \quad (2.29)
\end{align*}
Figure 2.4: (a) The real index of refraction $n(\omega)$ and (b) the absorption coefficient $\alpha(\omega)$ for a single gain or absorption line. Both quantities are plotted in natural units. Solid lines represent the exact function in Eq. 2.23 and the dashed lines represent a second order Taylor expansion.
For typical atomic vapors as we use in our experiments, $n_{\infty} = 1$, in which case the group velocity is solely determined by the ratio $\alpha_0/\gamma$. Because $\gamma$ is always positive, the sign is determined by $\alpha_0$. If $\alpha_0 < 0$ (gain), then $0 < v_g < c$ (slow light). If, instead, $\alpha_0 > 0$ (absorption), then $v_g$ will be fast. The width of the resonance $\gamma$ affects $v_g$ by increasing or decreasing the slope $dk(\omega)/d\omega$ for a given $\alpha_0$; a small $\gamma$ will lead to a sharper feature, and more extreme $v_g$, whether it be fast or slow.

The group delay $t_g$ can be found by combining Eqs. 2.25 and 2.19, which yields

$$t_g = \frac{L}{c}(n_{\infty} - 1) - \frac{\alpha_0 L}{2\gamma}. \quad (2.30)$$

When $n_{\infty} = 1$, the first term vanishes, and we are left with only the second term. Recall that the quantity $\alpha_0 L$ describes the total attenuation or amplification of light at the central frequency $\omega_0$, according to Eq. 2.22. Given a pulse of intensity FWHM $t_w$ and $n_{\infty} = 1$, the relative advancement is given by

$$A = -\frac{t_g}{t_w} = \frac{\alpha_0 L}{2\gamma t_w}. \quad (2.31)$$

In order to limit the distortion from higher-order terms in $k(\omega)$, we must limit the spectral bandwidth of the pulse. How severely we limit the bandwidth is an arbitrary choice, with shorter pulses leading to larger $A$ at the cost of greater distortion [31]. As an example, let us choose to set the power spectrum FWHM to $\gamma 2/5$. For a Gaussian pulse, this means $t_w \gamma = 10 \ln 2$. Inserting this into Eq. 2.31, we find

$$A = \frac{\alpha_0 L}{20 \ln 2} \approx 0.07\alpha_0 L. \quad (2.32)$$

With this choice of pulse width, we find that the relative advancement is completely determined by the total gain or absorption of the system. For the extreme values
of \( \alpha_0 L = \pm 15 \), which correspond to gain or attenuation by a factor of \( \approx 3.3 \times 10^6 \), we have \( A \approx \pm 1.1 \).

The second-order term in the expansion of \( k(\omega) \) will lead to compression or expansion according to Eq. 2.16, which shows that the pulse width is expanded by the factor \( f_{\text{exp}} \), where

\[
 f_{\text{exp}}^2 = \frac{t_f^2 - i k_2 L}{t_f^2} = \frac{t_f^2 - (\alpha_0 L) / \gamma^2}{t_f^2} = 1 - \frac{\alpha_0 L}{t_f^2 \gamma^2}.
\]

(2.33)

Because \( t_w \) and \( t_f \) are related by \( t_w = t_f 2\sqrt{\ln 2} \approx 1.67 t_f \), we then have

\[
 f_{\text{exp}}^2 = 1 - \frac{\alpha_0 L}{25 \ln 2} \approx 1 - 0.06 \alpha_0 L.
\]

(2.34)

Using the extreme value of \( \alpha_0 L = 15 \), we find that the pulse is compressed by a factor of \( f_{\text{exp}} \approx 0.3 \). That is, it emerges at only 30\% its original width. There are other ways to limit the distortion, of course. In the example above, we set the pulse width to some factor of \( \gamma \) so that both the advancement and distortion increase with increasing attenuation. It is possible to instead set \( t_w \) to a suitable multiple of \( \gamma / \sqrt{\alpha_0 L} \). This has the effect of fixing the second-order distortion. In that case, however, the magnitude of the advancement is no longer proportional to \( |\alpha_0 L| \), but to \( \sqrt{|\alpha_0 L|} \).

As stated above, it has been known for more than a century that the single absorption line allows fast light. However, one obvious problem is that the pulse is necessarily attenuated. Using the parameters above, a relative advancement of \( A = 0.5 \) requires attenuation by a factor of \( e^{-7.1} = 8 \times 10^{-4} \). In addition to making the detection of advanced pulses difficult, this attenuation often clouds their interpretation; the leading edge of the advanced pulse crosses any fixed intensity
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threshold later than a vacuum pulse simply because the advanced pulse is smaller. This effect is often erroneously cited as the reason causality is not violated in these experiments [32]. This attenuation is not a fundamental requirement for fast light systems, as will be shown in the next section.

2.3.2 Dual gain

In 1994, Steinberg and Chiao proposed a technique for creating fast light that uses a gain doublet, or pair of gain lines. In the previous section, gain led to slow light, but if the pulse is tuned to the spectral region between the gain lines, it will generally experience fast propagation. As in the previous section, the derivation here is valid for both gain and absorption, but this system was designed primarily for gain. Specifically, one can use Raman gain, which produces gain peaks with easily tuned positions and heights. When using this technique, it is possible to produce fast-light pulses with no attenuation. In fact, the pulses are slightly amplified.

If we assume that both lines have the same width $\gamma$ and line-center absorption coefficient $\alpha_0$, then the complex index of refraction can be written as

$$n(\omega) = n_\infty - \frac{c}{\omega} \alpha_0 \frac{\gamma}{2} \left( \frac{\gamma}{\omega - (\omega_0 - \delta/2) + i\gamma} + \frac{\gamma}{\omega - (\omega_0 + \delta/2) + i\gamma} \right), \quad (2.35)$$

where $\omega_0$ is the central frequency (and will be the pulse carrier frequency), $n_\infty$ is the background refractive index, and $\delta$ is the separation between the lines. Note the similarity between this and the $n(\omega)$ for a single line, as shown in Eq. 2.21. The wavenumber $k(\omega) = n(\omega) \omega/c$ can then be written as

$$k(\omega) = \frac{\omega}{c} n_\infty - \frac{\alpha_0}{2} \frac{\gamma}{2} \left( \frac{\gamma}{\omega - (\omega_0 - \delta/2) + i\gamma} + \frac{\gamma}{\omega - (\omega_0 - \delta/2) + i\gamma} \right). \quad (2.36)$$
Expanding this $k(\omega)$ as described in Sec. 2.1.2, leads to the following coefficients:

\begin{align*}
  k_0 &= \left( \frac{\omega_0}{c} n_\infty + i \alpha_0 \frac{\gamma^2}{(\delta/2)^2 + \gamma^2} \right), \quad (2.37) \\
  k_1 &= \frac{1}{c} \left( n_\infty + \alpha_0 \frac{\gamma}{c} \frac{(\delta/2)^2 - \gamma^2}{((\delta/2)^2 + \gamma^2)^2} \right), \quad (2.38) \\
  k_2 &= 2i\alpha_0 \gamma^2 \frac{3(\delta/2)^2 - \gamma^2}{((\delta/2)^2 + \gamma^2)^3}, \quad (2.39) \\
  k_3 &= 6\alpha_0 \gamma \frac{(\delta/2)^4 - 6(\delta/2)^2 \gamma^2 + \gamma^4}{((\delta/2)^2 + \gamma^2)^4}, \quad (2.40)
\end{align*}

Like the single-line case, coefficients after $k_0$ alternate between pure real and pure imaginary due to symmetry. In the case where $\delta$ can be easily varied, as is true in our experiments, one must choose a value for it. This is most easily visualized by substituting $X = \delta/2\gamma$ into Eqs. 2.37-2.40. Doing so yields

\begin{align*}
  k_0 &= \left( \frac{\omega_0}{c} n_\infty + i \alpha_0 \frac{1}{X^2 + 1} \right), \quad (2.41) \\
  k_1 &= \frac{1}{c} \left( n_\infty + \frac{\alpha_0 c}{\gamma} \frac{X^2 - 1}{(X^2 + 1)^2} \right), \quad (2.42) \\
  k_2 &= \frac{2i\alpha_0}{\gamma^2} \frac{3X^2 - 1^2}{(X^2 + 1)^3}, \quad (2.43) \\
  k_3 &= \frac{6\alpha_0}{\gamma^3} \frac{X^4 - 6X^2 + 1}{(X^2 + 1)^4}, \quad (2.44)
\end{align*}

Each of these fractions containing the quantity $X$ simply becomes a geometric factor
that is completely determined by the choice of $\delta/\gamma$. There are three logical choices:

$$\delta = \gamma 2\sqrt{3} \quad \rightarrow \quad \text{optimize } v_g; \quad k_1 = \frac{1}{c} \left( n_\infty + \frac{\alpha_0 c}{\gamma} \left( \frac{1}{8} \right) \right)$$  \hspace{1cm} (2.45)

$$\delta = \gamma \frac{2}{\sqrt{3}} \quad \rightarrow \quad k_2 = 0; \quad k_1 \equiv \frac{1}{c} \left( n_\infty + \frac{\alpha_0 c}{\gamma} \left( \frac{-3}{8} \right) \right)$$  \hspace{1cm} (2.46)

$$\delta = \gamma (2 + 2\sqrt{2}) \quad \rightarrow \quad k_3 = 0; \quad k_1 \equiv \frac{1}{c} \left( n_\infty + \frac{\alpha_0 c}{\gamma} \left( \frac{\sqrt{2} - 1}{4} \right) \right)$$  \hspace{1cm} (2.47)

Of these choices, the second produces slow light for negative $\alpha_0$ (gain), which is counter to the original purpose of the dual-gain-line technique. However, that does not mean that it is a useless choice. That choice ($\delta = \gamma 2/\sqrt{3}$) places the lines so close together that they effectively behave as a single line, but with the second order distortion eliminated. The first and third choices are actually quite similar and produce nearly identical results. For the remainder of this discussion, I will use the third choice, which results in $k_3 = 0$. Figure 2.5 shows $n(\omega)$ and $\alpha(\omega)$ for the third choice, $\delta = \gamma (2 + 2\sqrt{2})$, along with the index of refraction and absorption coefficients that correspond to the second order expansion of $k(\omega)$. Because $k_3 = 0$ this is also equal to the third-order expansion.

The group velocity for a double line, given this choice for $\delta/\gamma$, is given by

$$v_g = \frac{1}{k_1} = \frac{c}{n_\infty + \frac{\alpha_0 c \sqrt{2} - 1}{4}}.$$  \hspace{1cm} (2.48)

Like in the single gain line case, we can often assume $n_\infty = 1$, and so the group velocity is again determined by the ratio $\alpha_0/\gamma$. In this case, the sign is reversed, so that if $\alpha_0 < 0$ (gain), then $v_g$ will be fast. If $\alpha_0 > 0$ (absorption), then $v_g$ will be slow.
Figure 2.5: (a) The real index of refraction \( n(\omega) \) and (b) the absorption coefficient \( \alpha(\omega) \) for a double gain or absorption line. Both quantities are plotted in natural units. Solid lines represent the exact function in Eq. 2.23 and the dashed lines represent a second order Taylor expansion. This is plotted with the choice \( \delta = \gamma(2 + 2\sqrt{2}) \).
The group delay \( t_g \) can be found by combining Eqs. 2.38 and 2.19, which yields

\[
 t_g = \frac{L}{c} (n_\infty - 1) + \frac{\alpha_0 L \sqrt{2} - 1}{\gamma}. \tag{2.49}
\]

When \( n_\infty = 1 \), the first term vanishes, and we are left with only the second term. Because the pulse’s carrier frequency is now between the gain lines, the carrier frequency is no longer amplified or attenuated by \( e^{-\alpha_0 L} \), but by

\[
e^{-3[2k_0]L} = e^{-\frac{\alpha_0 L}{2 \sqrt{2}}} \approx e^{-0.3 \alpha_0 L}. \tag{2.50}
\]

Given a pulse intensity FWHM of \( t_w \) and \( n_\infty = 1 \), the relative advancement is given by

\[
 \mathcal{A} = -\frac{t_g}{t_w} = -\frac{\alpha_0 L \sqrt{2} - 1}{\gamma t_w}. \tag{2.51}
\]

Like in the single line case, we must limit the spectral bandwidth of the pulse in order to limit the distortion from higher-order terms in \( k(\omega) \). Precisely how we limit the spectral bandwidth is again an arbitrary choice, with shorter pulses leading to larger \( \mathcal{A} \) at the cost of greater distortion [31]. In this case, the region of smooth linear dispersion is wider, so we can choose a larger pulse bandwidth without sacrificing distortion. As an example, let us choose to set the power spectrum FWHM to \( \delta/6 = \gamma(1 + \sqrt{2})/3 \approx 0.8 \gamma \). For a Gaussian pulse, this means \( t_w \gamma = (12 \ln 2)/(1 + \sqrt{2}) \). Inserting this into Eq. 2.31, we find

\[
 \mathcal{A} = -\frac{\alpha_0 L}{48 \ln 2} \approx -0.03 \alpha_0 L. \tag{2.52}
\]

With this choice of pulse width made, we find that the relative advancement is again completely determined by the total gain of the system. For the extreme value
of $\alpha_0 L = -15$, which corresponds to a pulse amplification factor of $\approx e^{4.5} \approx 90$, we have $A \approx 0.45$.

In the gain doublet case, $k_2$ leads to compression according to Eq. 2.16. The pulse compression factor is $f_{\text{exp}}$, where

$$f_{\text{exp}}^2 = \frac{t_f^2 - ik_2 L}{t_f^2} = 1 + \frac{\sqrt{2} - 1}{4} \frac{\alpha_0 L}{t_f^2 \gamma^2} \approx 1 + 0.1 \frac{\alpha_0 L}{t_f^2 \gamma^2}. \quad (2.53)$$

Because $t_w$ and $t_f$ are related by $t_w = t_f 2\sqrt{\ln 2} \approx 1.67 t_f$, we then have

$$f_{\text{exp}}^2 = 1 - \alpha_0 L \frac{1 + \sqrt{2}}{144 \ln 2} \approx 1 - 0.024 \alpha_0 L. \quad (2.54)$$

Consider once again our extreme example, with $\alpha_0 L = -15$; we find that the pulse is compressed by a factor of $f_{\text{exp}} = 0.8$, or reduced to 80% of its original width. As in the single-line case, it is possible to choose other distortion-limiting schemes that will result in quantitatively different results or different scaling, but this provides a general overview of the issues.

Compared to the single absorption line, the dual-gain technique presents some interesting trade-offs. It is possible to achieve significant pulse advancement with relatively little distortion, and a fairly small amount of pulse amplification.\(^3\) The biggest drawback is that there must be two nearby gain lines. The very presence of these two large gain lines can result in competing nonlinear effects, as described in Appendix A.

\(^3\)The line-center gain is much larger, of course.
2.3.3 Other fast- and slow-light techniques

There are many other techniques for generating fast and slow light that have been used by other researchers. Most (if not all) are mathematically and physically equivalent to the techniques described above, although they are implemented differently [11]. Also, the techniques presented here are not intended to form an exhaustive list of the research in this field. They are only highlights of some of the techniques.

Most of the well-publicized recent slow light experiments have used a technique called electromagnetically-induced transparency (EIT). This phenomenon is based on quantum interference effects [33]. EIT can be used to produce extremely narrow spectral regions of transparency in a region where light is otherwise absorbed. Light pulses tuned to this narrow transparency window will be transmitted with slow group velocities. The reason the width of these features can be made so small is that they are related to the coherence lifetime between stable ground states of an atom rather than the lifetimes of excited states, as is usually the case for simple gain or absorption. Another advantage over the simple gain line technique is that the central absorption coefficient is approximately zero, meaning the pulse is largely unchanged in height. Recent experiments using this technique have achieved group velocities of only a few meters per second [8] and ultimately even stopped light [9,10].

A family of common techniques for creating fast light is based on various forms of quantum tunnelling. Examples include frustrated total internal reflection [34,35] and waveguide propagation below the cutoff frequency [36]. Similar experiments can also be performed by propagating pulses through multiple layers of varying dielectrics [37]. This creates frequency-dependant transmission properties similar to those of the gain or absorption techniques [38,39]. Similar results have also been
achieved in electrical pulse transmission through coaxial photonic crystals [40].

A final group of techniques is perhaps the most novel. I refer to these as “geometric techniques” because they rely on the geometry of the propagating waves rather than the properties of the medium. In fact, these techniques are often employed in vacuum. One example is the use of an axicon (a cone-shaped lens or mirror) to focus a beam to a line [41]. A pulse can be transmitted through an axicon such that the peak of the pulse measured on-axis after the axicon moves at velocities faster than \( c \). In this case, one is interfering rays that are traveling in different directions. Similar results have been achieved with microwaves in concave reflection [42].
Chapter 3

Information on fast and slow light pulses

In the previous chapter, I described the theory of fast and slow light along with several techniques for generating fast and slow light. However, I did not address the issue of information encoded on fast- or slow-light optical pulses. In this chapter, I will focus on the velocity of information on optical pulses.

The question of how fast information on an optical pulse travels is an important one for many reasons. One of the original reasons is to address the question of Einstein causality; can information on fast pulses move faster than \( c \)? If so, is relativity wrong? Can the principle of causality (cause must come before effect) be violated? Most researchers, including myself, are sufficiently confident in both Einstein’s relativity and the principle of causality that they would be very surprised if information could travel faster than \( c \). Nonetheless, it is a fascinating question due to its broad philosophical implications. It also serves as a useful test of the theoretical and experimental tools for measuring \( v_i \); if we find \( v_i \) faster than \( c \), then we are wrong about something. We could be wrong in trusting relativity, but our confidence in relativity forces us to look closely at our techniques for measuring \( v_i \). However, I must emphasize that it would be wholly inappropriate to devise theoretical and experimental techniques specifically to produce \( 0 < v_i \leq c \) for the
CHAPTER 3. INFORMATION ON FAST AND SLOW LIGHT PULSES

sake of relativity; relativity must stand on its own and endure (or fail to endure) any scientific discoveries made in this field.

Another major reason for interest in the information velocity is a more practical one. As a society, we have become more and more dependent on the transmission and processing of information, and we increasingly turn to optical techniques for these tasks. Furthermore, as our needs grow, we are pushing the boundaries of our technological devices and knowledge. We are optimizing nearly every aspect of information-handling. It can only serve us well to have a better understanding of precisely how information is transmitted on optical pulses.

3.1 Relativistic implications

It is generally agreed that Einstein’s theory of special relativity, combined with the principle of causality, limits the speed of information to $c$, the speed of light in vacuum. This is independent of the transmission mechanism. It applies equally well to optical pulses, electronic pulses, and printed copies of this thesis zooming across the universe in a space ship.

If we focus on light pulses and consider the specific case of negative group velocities, as discussed in Sec. 2.2, we can understand the concern for causality even without relativity! Imagine for a moment that a pulse can be transmitted perfectly without distortion at a negative group velocity and that information really does travel at that group velocity. In that case, the receiver, at one end of the medium, could receive the pulse perfectly before the sender, at the other end of the medium, had sent it. This can very quickly lead to any number of paradoxes. For example,

\[\text{If a pulse could be transmitted perfectly without distortion at any velocity, then information would have to travel at that velocity. What other velocity could it travel at?}\]
what if one of the sender’s possible messages is “tell me to send something else” or even simply “kill me immediately.” Clearly, this would be strange!

As stated, the example above seems absurd, but it provides a clear connection to the relativistic implications of faster-than-$c$ information transmission. Let us begin by considering information that is sent at time $t = t_0 = 0$ from position $z = 0$, and that arrives at position $z = L > 0$ at time $t_1$. If $t_1 < 0$, then we are trivially back to the absurd case described above. We now have the information velocity

$$v_i = \frac{L}{t_1}. \quad (3.1)$$

All of these quantities are as measured in the rest frame $S$. Now, imagine an observer in frame $\tilde{S}$ moving in the $\hat{z}$ direction at velocity $v_R$, which could be negative. If we transform our measurements into the $\tilde{S}$ frame according to the Lorentz transformations (choosing the origin of $\tilde{S}$ so that the information is sent at $\tilde{t} = 0, \tilde{z} = 0$), then we find [43]

$$\tilde{L} = \gamma_R (L - v_R t_1), \quad (3.2)$$

$$\tilde{t}_1 = \gamma_R \left( t_1 - \frac{v_R}{c^2} L \right), \quad (3.3)$$

where $\gamma_R$ is the relativistic scale factor. Combining these two equations, we find
that the velocity of information in $\bar{S}$ is given by

$$\bar{v}_i = \frac{\bar{L}}{\bar{t}_1},$$

$$= \frac{\gamma_R (L - v_R t_1)}{\gamma_R (t_1 - \frac{v_R L}{c^2})},$$

$$= \frac{L}{t_1 - v_R} - \frac{v_R L}{1 - \frac{v_R^2}{c^2}},$$

$$= \frac{v_i - v_R}{1 - \frac{v_R^2}{c^2}}.$$  \hspace{1cm} (3.4)

Here, we have effectively re-derived Einstein’s velocity addition rule. Now, consider $v_i > c$. For any $v_R$ that satisfies $-c \leq v_R \leq c$, the numerator of Eq. 3.4 will be positive. However, the denominator will be negative if $v_R/c > c/v_i$. Because we have already stated that $v_i > c$, this can be satisfied by a $v_R$ such that $v_R < c$. Therefore, if the observer is traveling at $v_R > c^2/v_i$, she will observe the information propagate with negative velocity! Again, we are back to the case described above. If $v_i$ were not faster than $c$, the observer could not observe $v_i < 0$ unless she were moving faster than $c$ herself.

In short, negative information velocities would violate the principle of causality, a fact that does not depend on relativity. The role of relativity is to allow $v_i > c$ to appear as a negative velocity to some observer, who need not travel faster than $c$ herself. Therefore, relativity tells us that $v_i > c$ will also violate causality.

### 3.2 Overview of information velocity research

A great deal of theoretical and experimental research has been devoted to the topic of fast- and slow-light pulses, and to the velocity of information encoded on them. Because a number of excellent review articles have been published recently on fast
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and slow light [11, 18, 20], I will only discuss here those results which specifically deal with the velocity of information.

3.2.1 The signal velocity

Any discussion of information velocity research should begin with the work of Sommerfeld and Brillouin. Not only is their work significant from a historical perspective, but it continues to dominate the field. Many modern researchers believe that Sommerfeld and Brillouin’s theoretical results are both correct and complete, that there is nothing more to be done [4,44]. Others feel that, although Sommerfeld and Brillouin’s work is tremendously useful and powerful as an analysis of pulse propagation in dispersive media, it does not fully address the question of information velocity [11].

At the beginning of the 20th century, the scientific community was well aware that the group velocity could exceed $c$ in anomalously-dispersive media. With the arrival of Einstein’s theory of relativity, they became concerned about this apparent conflict. It was this concern that led Sommerfeld and Brillouin to explore pulse propagation in dispersive media. As discussed in Sec. 2.1, the group velocity is only a mathematical approximation, and cannot perfectly describe the behavior of complex pulses in complex media. A much more careful study of pulse propagation was needed in order to adequately address the issue. To fill this need, Sommerfeld and Brillouin performed a careful and thorough theoretical analysis of the propagation of square pulses through anomalously dispersive media.

Sommerfeld and Brillouin’s research showed that square pulses propagating through an anomalously-dispersive medium become badly distorted. They found that pulses separate into three parts or regions. The first to arrive is the group
of first precursors, or forerunners, which are high-frequency fluctuations that bear little resemblance to the original pulse. They found these precursors, sometimes called the Sommerfeld precursors, to move at the speed $v_f = c$. Being the first disturbance associated with the original pulse, this is also the speed of the pulse front. The front is the first point where the field deviates from zero. Next comes a second set of precursors (Brillouin precursors), which moves slower and is composed of lower frequencies. Finally, the main body of the pulse comes. In normal dispersion, they found that the velocity of the main body of the pulse is well-described by the group velocity, but in anomalous dispersion, it is not. In anomalous dispersion, the group velocity is faster than $c$, but the main body of the pulse moves slower than $c$. Based on these findings, Sommerfeld and Brillouin set out to define the signal velocity $v_s$.

It is at this point that Sommerfeld and Brillouin diverged somewhat. Because Brillouin believed the front and precursors to be very small, he expected them to be generally undetectable and to contain no useful information. Therefore, he felt strongly that the signal velocity should be the velocity of the main pulse body. For normally dispersive media, where the group velocity accurately describes the motion of the main body, he wanted $v_s = v_g$. However, near anomalous dispersion, where $v_g$ exceeds $c$ and the pulse appeared to travel slower than $c$, he wanted $v_s$ to represent the speed of the main body, and therefore to deviate from $v_g$.

In order to achieve this goal of having $v_s$ describe the motion of the main body of the pulse and to degenerate to $v_s = v_g$ away from anomalous dispersion, Brillouin defined $v_s$ in several different and progressively more complicated ways. The first and most well-known definition of $v_s$ is simply the velocity of the point at which the pulse first reaches half its maximum height. Sommerfeld correctly pointed out
and Brillouin acknowledged) that this definition is largely arbitrary, and in the limit that the threshold is reduced, \( v_s \rightarrow v_f = c \). Brillouin also created several more mathematically rigorous definitions based on such techniques as the method of stationary phase, saddle point integration, and Fourier analysis. Nonetheless, even with these more mathematically sound definitions, Brillouin acknowledges the arbitrariness of his signal velocity, writing

In general, the signal velocity measured depends on the sensitivity of the detecting apparatus used. With a sufficiently sensitive detector, even the forerunners, or certain parts of them, might be detected, and the resulting measurement would imply a very large velocity of propagation. But if the sensitivity of the detector is restricted to a quarter or half the final signal intensity, then an unambiguous definition of the signal velocity can, in general, be given.

Much of the arbitrariness of the original definitions of \( v_s \) have been removed with subsequent work by other researchers [45], but the fundamental problem remains: The signal velocity is not the velocity of information. Consider this simple example. Imagine a sender with a very powerful pulse generator and a receiver with a very sensitive detector. If the sender and receiver have agreed that a pulse will be sent immediately upon the occurrence of some important event, then the receiver need only await the arrival of any electromagnetic wave at all. The receiver need not know what the main body of the pulse will look like. It is enough that he knows a pulse has been sent. Therefore, from this simple example, we see that information can travel faster than the signal velocity.

To be fair, it seems quite likely that both Sommerfeld and Brillouin fully understood that the signal velocity is not the velocity of information as we mean today. It
is my suspicion that they, being convinced that relativity was safe because $v_f = c$, intended for $v_s$ to be a useful practical description of a general pulse shape, not a rigorous upper-bound on information transmission. Unfortunately, their choice of the label “signal” for this velocity seems to have forever linked it with the concept of information transfer.

3.2.2 Recent research

Since the work of Sommerfeld and Brillouin, there have been many different and often conflicting experimental and theoretical results [7, 11, 12, 16, 17, 34, 45–53]. While most (but not all, see Refs. [34, 46]) of these results argue that relativistic causality is not threatened, the explanations vary wildly.

For example, Nimtz and Haibel claim that all information-bearing waveforms have non-zero temporal extent and, while these waveforms can be superluminally advanced, causality is preserved because the advancement must be less than the extent of the entire waveform [16]. However, this claim seems to rest on the assumption that no information is available until the entire waveform has been received, which is contrary to the way most modern communication systems function.

In contrast, Chiao and coworkers suggest that information is encoded in points of non-analyticity, which cannot be superluminally advanced [11]. In fact, they claim that these non-analytic points move at precisely $c$ in any medium because they necessarily have frequency components extending to infinity, and because the refractive index $n(\omega) \to 1$ as $\omega \to \infty$ for any medium. Nimtz and Haibel object to the suggestion of frequencies extending to infinity, and therefore to the concept of a true physical discontinuity [16]. There is also the question of how a “point” on a waveform can be detected; it contains no energy itself so there is nothing to detect.
Chiao’s theory is discussed in more detail in Sec. 3.4.

In a related paper, Kuzmich et al. demonstrate that points on a waveform with a constant signal to noise ratio (SNR) move no faster than $c$, although they present no strong connection between these points and information [51].

### 3.2.3 Information on slow-light pulses

The question of the information on slow-light pulses is even less clear; there has been little research devoted to this case. It is largely believed that, when $0 < v_g \leq c$, $v_s \approx v_g$ and hence the group velocity $v_g$ describes the propagation of information. This belief can be seen in many texts [54,55] and articles [50]. In contrast, Chiao and coworkers claim that information also travels at $c$ in slow-light media, for precisely the same reason as in the fast-light case.

### 3.2.4 Measurement of the information velocity

The research described above is mostly theoretical and has largely attempted to demonstrate what the information velocity is not; it is not greater than $c$. Where experiments have been performed, little effort has been made to measure the velocity of information [7]. One notable exception is the research by Centini et al., in which they measured what they called the “operational signal velocity” by detecting the time when the pulse intensity crosses some threshold [17]. They find that their operational signal velocity is never faster than the group velocity. Unfortunately, they do not provide a clear link between this operational signal velocity and information.

There is as yet no accepted technique for measuring the information velocity. However, any such technique should satisfy two important criteria. First, it should
be truly information-based. That is, there should be a clear mathematical and physical link between the measured quantity (a power, voltage, etc.) and information. Second, the measurement technique should be independent of the medium. If the pulse front is measured for fast-light pulses, then it should also be measured for slow-light pulses. In Ch. 5, I present a technique that I believe satisfies both of these criteria.

### 3.3 The need for large advancement or delay

Regardless of the specific theoretical prediction or physical interpretation, a practical requirement for measuring the information velocity is large pulse advancement or delay. The reason is simple; in the laboratory, researchers tend not to measure velocities directly, but rather to measure positions and times. Even if two pulse features travel at radically different velocities, the difference between the velocities may not be clear if they do not propagate far enough at those velocities. For example, if we wish to test whether a feature moves at the group velocity $v_g$ or $c$, it is not enough that $v_g$ be very different from $c$; we must ensure that the group delay $t_g$ be very different from zero. In practice, the precise value of “very different” is determined by the resolution and noise characteristics of the apparatus, as well as the temporal extent of the features in question.

### 3.4 Information in non-analytic points

As described above, Chiao and collaborators have proposed that all transmitted information is contained in non-analytic points on a waveform. They claim that if an analytic function is known over any finite (but arbitrarily small) interval, it is
knowable at all points. Therefore, if a waveform consists of a front, after which it is truly analytic, then everything there is to know about the waveform can be known immediately after the front arrives. However, if there is a point of non-analyticity later in the waveform, that which lies beyond the non-analytic point cannot be known. As a result, Chiao and coworkers claim that the velocity of information is equal to the velocity of these points of non-analyticity. They also claim that non-analytic points are essentially fronts, so that \( v_i = v_f = c \).

### 3.5 The front velocity

Because the front velocity \( v_f \) is such an important quantity, and Chiao’s theory relies on it so heavily, it seems appropriate to present a simple proof that \( v_f = c \). There are many different ways to show that a discontinuity (including a pulse front) moves at speed \( c \) \([2, 4, 5]\), but I present an exceptionally short and clear one here. This proof was sent to Brillouin by Dr. T. Levi-Civita, and is printed (in a considerably more compact form) in Brillouin’s book \([2]\).

Let us begin with a model for the interaction between the light and matter. Let us imagine a series of charged particles of mass \( m \), charge \( e \), and density \( n_p \) displaced from their equilibrium positions by distance \( s \) in the \( x \) direction. The light propagates along the \( z \) axis and is linearly polarized in the \( x \) direction so that we can describe it by its electric field in the \( x \) direction \( \mathcal{E} \) and its magnetic field in the \( y \) direction \( \mathcal{B} \). The interaction is then described by
Figure 3.1: An example of a discontinuous function $f(x)$ and the value of the discontinuity operation $[f]$. 

\[
\begin{align*}
\frac{1}{c} \frac{\partial B}{\partial t} &= - \frac{\partial E}{\partial z}, \\
\frac{1}{c} \left( \frac{\partial E}{\partial t} + n_p c \frac{\partial s}{\partial t} \right) &= - \frac{\partial B}{\partial z},
\end{align*}
\]

which are simply Maxwell’s equations. This model is clearly only approximate, but it is still a reasonable simple model for the interaction between a propagating field and matter.

Let us now imagine that the front of the wave has continuous $E$ and $B$, but discontinuous $\partial E/\partial t$ and $\partial B/\partial t$. Let us further define the bracket notation $[f]$ to be the height of the discontinuity in some function $f$, as shown in Fig. 3.1. If the function $f$ is not discontinuous, then $[f] = 0$. We will use this notation to represent the heights of the discontinuities in $E$, $B$, and their partial derivatives. Using this
new notation, let us define

\[ e_d \equiv \frac{\partial \mathcal{E}}{\partial z} = -\frac{1}{v_f} \frac{\partial \mathcal{E}}{\partial t} \quad \text{and} \quad h_d \equiv \frac{\partial \mathcal{B}}{\partial z} = -\frac{1}{v_f} \frac{\partial \mathcal{B}}{\partial t}, \tag{3.7} \]

where \( v_f \) is the front velocity. This relationship between the discontinuity heights in the \( z \) and \( t \) derivatives can be trivially demonstrated if \( \mathcal{E} \) and \( \mathcal{B} \) are of the form \( f(z,t) = f(t'(z,t)) \) with \( t'(z,t) = t - z/v_f \). However, it can be shown much more generally as follows;

\[
\begin{align*}
\mathcal{E}_-(z + \delta z, t + \delta t) - \mathcal{E}_-(z, t) &= \frac{\partial \mathcal{E}_-}{\partial t} \delta t + \frac{\partial \mathcal{E}_-}{\partial z} \delta z, \\
\mathcal{E}_+(z + \delta z, t + \delta t) - \mathcal{E}_+(z, t) &= \frac{\partial \mathcal{E}_+}{\partial t} \delta t + \frac{\partial \mathcal{E}_+}{\partial z} \delta z, \\
0 &= \left[ \frac{\partial \mathcal{E}}{\partial t} \right] \delta t + \left[ \frac{\partial \mathcal{E}}{\partial z} \right] \delta z. \tag{3.8}
\end{align*}
\]

The first two of these are the same equation evaluated immediately before and after the discontinuity. The third is simply the difference of the first two, noting that \([\mathcal{E}] = 0\) because the electric field is continuous. If we define \( v_f \equiv \delta z/\delta t \), then we arrive at the relations in Eq. 3.7. Clearly, the same procedure can be applied to the magnetic field as well.

If we perform the same operation on the Eqs. 3.5 and 3.6 and use \([\partial s/\partial t] = 0\), we find

\[
\begin{align*}
\frac{1}{c} \left[ \frac{\partial \mathcal{B}}{\partial t} \right] &= -\left[ \frac{\partial \mathcal{E}}{\partial z} \right], \\
\frac{1}{c} \left[ \frac{\partial \mathcal{E}}{\partial t} \right] &= -\left[ \frac{\partial \mathcal{B}}{\partial z} \right], \tag{3.9}
\end{align*}
\]

which can be multiplied together to yield

\[
c^2 = \left[ \frac{\partial \mathcal{B}}{\partial t} \right] \left[ \frac{\partial \mathcal{E}}{\partial z} \right] = \frac{(-v_f h_d)(-v_f e_d)}{h_d e_d} = v_f^2. \tag{3.10}
\]

We have now shown that \( v_f = c \) in any medium and for any propagating electro-
magnetic wave. Note that we nowhere in this proof did we assume that the field was zero before the discontinuity. Therefore, this applies equally well to the propagation of a discontinuity on an otherwise continuous and finite field. This derivation only considers discontinuities in the first derivative, but the same result has been shown for all derivatives [11, 12].
Chapter 4

Preparation of fast- and slow-light media

To measure the information velocity in fast- and slow-light media, it is first necessary to prepare suitable media. Ideally, such media would be capable of generating large positive and negative group delays as described in Ch. 2 without excessive pulse distortion. In this chapter, I describe the creation of two such media—one fast and one slow—and a numerical model used to predict and better understand the propagation of pulses through these media. Both media are based on Raman gain using the techniques described theoretically in Sec. 2.3. The slow-light medium is based on a single Raman gain line and the fast-light medium is based on a Raman gain doublet.

4.1 A gain-based fast-light medium

The fast-light medium I use for my information velocity experiments is based on the gain doublet technique described in Sec. 2.3. This technique was first proposed by Steinberg and Chiao [26] and first implemented by Wang et al. [7]. It uses Raman gain to generate the two gain lines. Unlike Wang et al., I use two separate cells to generate the two gain peaks in order to avoid the competing nonlinear effects
described in Appendix A.

4.1.1 Raman amplification

The Raman scattering process is useful for these fast- and slow-light media because it is a very flexible technique for creating gain lines. I chose to use Raman amplification for the present experiments because our research group has many years of experience using Raman techniques [21, 22, 24, 25]. In that sense, this research builds off of the past research and knowledge acquired by this group about Raman processes.

Raman scattering is a nonlinear process in which two fields interact with an atom simultaneously such that one photon is created and another annihilated. In the process, the atom changes energy levels [30]. It is conventional to refer to the two fields as the probe and pump (or drive) fields. Usually, the probe is the field of interest and weak, whereas the pump is powerful and interesting only due to its effects on the atoms and—indirectly via the atoms—the probe. In the present context, the probe refers to the information-carrying optical pulse.

Figure 4.1 shows several simple level diagrams of Raman transitions. In each of them, $\omega_p$ is the probe frequency, $\omega_d$ is the pump frequency, and $\Delta_R$ is the detuning between the virtual Raman level and a nearby real energy level of the atom. The first process, shown in (a) is Raman absorption. In analogy to normal absorption, a single probe photon is annihilated, contributing its energy to the atom and to the newly-created pump photon. The second process (b) is spontaneous Raman scattering, wherein a pump photon is annihilated and a probe photon is created. Just as in normal spontaneous emission, the created photon will emerge in an unknown direction. Note that these first two processes are effectively the same process, with
the roles of pump and probe reversed. In the example shown, the probe has a higher frequency, but this is arbitrary. The final process (c) is stimulated Raman scattering, in which the atom interacts with incoming probe and pump photons simultaneously, and generates a new probe photon at the cost of the pump photon. Just like normal stimulated emission, the new photon is a clone of the original probe photon. It is this final process which is responsible for producing the gain lines in the setup described below.

It can be shown that stimulated Raman scattering will lead to a contribution to the absorption coefficient of the form

$$\alpha_{\text{Raman}}(\omega) \propto -\frac{I_d}{\Delta_R^2} \frac{\gamma_R}{(\omega - \omega_p)^2 + \gamma_R^2}$$

(4.1)

where $\gamma_R$ is the width of the Raman gain line [56]. This has precisely the same functional form as the absorption coefficients described in Sec. 2.3. The strength of the gain is proportional to the pump intensity $I_d$ and inversely proportional to the square of the detuning from the real energy level $\Delta_R$.

We see from Eq. 4.1 that a Raman gain line can be created near a real transition (detuned by $\Delta_R$) with tunable frequency and gain strength. To choose the frequency of the gain line, one must only set the frequency of the pump $\omega_d$. Then, one can achieve the desired gain strength by setting the pump intensity $I_d$. The only parameter that is not conveniently adjustable here is the width of the gain line $\gamma_R$. In the present context, this is the parameter which will be used to set the other frequencies and timescales in the experiments.
Figure 4.1: Simple Raman scattering diagrams for (a) absorption, (b) spontaneous scattering, and (c) stimulated scattering. In each case, the dashed lines represent the pump field of frequency $\omega_d$ and the solid lines represent the probe field of frequency $\omega_p$. 
4.1.2 The two-zone concept

The stimulated Raman scattering or Raman gain process can be used to create a customizable gain line as described above. In fact, because the resulting gain line is identical to a standard linear amplifying resonance (at least from the probe’s perspective), the technique can be used to create several gain lines. To create many gain lines, one need only introduce multiple pump beams, each with its own frequency. Unfortunately, these multiple pump beams, which are often quite intense, can interact with each other via the atoms and produce unwanted nonlinear effects. These effects are described in more detail in Appendix A.

The simplest way to avoid these competing nonlinear effects is to simply not allow multiple pumps to interact with the same atoms. This can be accomplished by sending the pumping beams through physically distinct regions of atoms. The probe can then be transmitted through each region in series. The obvious objection is that one is not truly creating a single medium with multiple gain lines; one is creating multiple media, each with a single gain line. While this is true, the two are equivalent as long as the gain processes are linear in the probe field. In that case, the effects of each region can simply be added together, even though they occur in different physical locations. Because all of the fast- and slow-light physics discussed in Ch. 2 is linear in the pulse/probe field, this is a completely adequate solution.

4.1.3 Experimental fast-light system

The experimental system used in my fast-light experiments is shown in Fig. 4.2. The setup is divided logically and physically into three stages. In stage 1, the beams are created and the pumping beams are set to the desired power and frequency. In stage 2, the probe beam is set to the desired frequency and can also be pulsed. In
stage 3, all three beams are combined in two vapor cells, and the resulting fast-light pulses are detected.

The first stage begins with the Coherent 899 titanium sapphire ring laser. This single laser is used to generate all of the beams in the experiment. A small fraction of the total power is immediately split off by a glass plate and injected into an optical fiber. This beam will ultimately become the probe beam. The remaining power is then attenuated by a variable attenuator and divided into two beams. The ratio between the powers in the two beams can be adjusted using the half-wave (λ/2) plate and polarizing beam-splitter (PBS) combination; the half-wave plate can be used to set the polarization to any arbitrary linear polarization. These two beams are then shifted in frequency by two acousto-optic (AO) deflectors. The resulting beams are then injected into two optical fibers for transport to stage 3.
At this point, the two pump beams have been set to the desired frequencies (by the AOs) and to the desired power (by the attenuator, $\lambda/2$, and PBS) to create a pair of Raman gain peaks as described in the previous section.

In stage 2, the probe beam is set to the desired frequency and pulsed. This is accomplished by using a double-pass AO (Isomet 1206C). The double-passing is achieved by retro-reflecting the beam after the AO through a quarter-wave plate. This results in an overall rotation of the polarization by $\pi/2$. Therefore, the input beam passed through the PBS, but the return beam will be reflected by it. The appeal of double-passing the AO is that any angular deflection introduced by the AO on the first pass is removed on the second pass. This allows the frequency of the AO to be scanned with little effect on the power and direction of the beam. Frequency-scanning of the beam is useful because it enables measurement of the gain as a function of probe frequency. The AO can also be modulated by an arbitrary waveform generator (AWG, Stanford DS345) in order to generate pulses. That is, the AO diffraction efficiency can be modulated to create arbitrary optical pulse shapes. The resulting probe beam is then inserted into another optical fiber.

In stage 3, the linearly polarized probe beam of frequency $\omega_0$ (which may be pulsed) propagates through two uncoated pyrex vapor cells of $^{39}$K and is detected by a photoreceiver (New Focus 1801-FS-AC). Each cell is 20 cm (= $L/2$) long and heated to 100° C to create a $^{39}$K atomic number density of $4.5 \times 10^{11}$ cm$^{-3}$. For the purpose of calculating velocities, the two cells can be treated as a single cell of length $L = 40$ cm. Before the first cell, a polarizing beam splitter is used to combine the probe with the orthogonally-polarized continuous-wave Raman pumping beam of frequency $\omega_{d_-}$. Between the cells, another PBS is used to separate the first pump from the probe and to combine a second pump beam of frequency $\omega_{d_+}$ with the
probe. After the second cell, a final PBS is used to separate the probe from the second pump.

The frequencies of the beams are shown in Fig. 4.3. The two pump beams create Raman gain resonances at $\omega_{d} + 462$ MHz, the ground state splitting of $^{39}$K. The frequencies of the beams are adjusted so that $\omega_{d-}$ is 1.36 GHz to the high-frequency side of the center of the $^{39}$K $4S_{1/2} \leftrightarrow 4P_{1/2} (D_1)$ transition and $\omega_{d+} = \omega_{d-} + 23$ MHz. Because the excited state splitting in $^{39}$K is 58 MHz, this implies that $\omega_{d-}$ is 1.62 GHz greater than the lowest-frequency ($F = 2 \rightarrow F' = 1$) transition. The probe beam is tuned to $\omega_0 = (\omega_{d-} + \omega_{d+})/2 + 462$ MHz, where there is anomalous dispersion and the associated fast group velocity.

Because the pumps are tuned to the high-frequency side of the $D_1$ line, they
also serve as optical pumps, preferentially populating the upper ground state. This
optical pumping occurs because atoms in the lower ground state are more likely
to absorb a pump photon and be excited to the excited states. Once there, atoms
are equally likely to decay to either ground state. This ground state inversion is
necessary for the Raman gain process. By tuning the laser itself, all beams can be
simultaneously tuned far from the potassium resonance so that the beams do not
interact with the potassium atoms and the cells are equivalent to vacuum. This is
how all “vacuum” data is acquired.

Figure 4.4a shows the experimentally measured gain profile (dots) for the fast-
light system. Because of the small asymmetry in the gain profile, optimum ad-
vancement is achieved by tuning to the region with the flattest gain, which is not
precisely between the lines. Also shown is a fit of this profile to a double-Lorentzian.
While the fit does not capture the details of the profile well (for example, it sug-
gests that the minimum gain occurs at a frequency approximately 2.5 MHz higher
that the true minimum), it is useful for qualitative predictions of (for example) the
group index, as shown in the next figure. Panels b and c of Fig. 4.4 show the pulse
spectra corresponding to the Gaussian pulses and to the pulse symbols (described
further in the following chapters). These spectra are normalized to unit height for
convenient comparison and correspond to the smoothed pulses described in Sec. 4.3.
These spectra are calculated by computing the square magnitude of the Fourier-
transformed electric field. Note that the Gaussian and “1” have nearly identical
full widths at half maximum (FWHM) because they have similar widths, while the
temporally shorter “0” symbol has a broader spectrum. Figure 4.5a shows the pre-
dicted group index \((c/v_g)\) using the fit in Fig. 4.4a. Note that the vast majority of
the pulse spectral power falls within the fast-light region where \(c/v_g < 1\).
Figure 4.4: Panel (a) shows the experimentally measured gain profile for the fast-light system (dots) and a calculated fit to a double-Lorentzian (line). Panels (b) and (c) show the power spectra of the Gaussian pulse (solid), “0” symbol (dashed) and “1” symbol (dotted).
Figure 4.5: Panel (a) shows the predicted group index \((c/v_g)\) using the fit in Fig. 4.4a. The horizontal line is drawn at \(c/v_g = 1\) and separates fast-light (below) and slow-light (above) regions. Panels (b) and (c) are identical to those in Fig. 4.4.
4.1.4 Experimental fast-light observations

Figure 4.6 shows the temporal evolution of 263-ns-long (FWHM) Gaussian pulses through both the fast-light medium and through vacuum. The peak power for the vacuum pulse is 21.1 $\mu$W and for the fast-light pulse is 23.2 $\mu$W. However, because the injected fast-light pulse is weaker by a factor of $10^{0.8} = 6.31$, this corresponds to pulse amplification by a factor of $6.94 = e^{1.94}$. These pulse powers are chosen both to avoid nonlinear effects such as gain-saturation, and also to keep the signal-to-noise ratio (SNR) similar for both cases. Because the detector is the primary noise source and the amount of noise is fixed, similar pulse powers lead to similar SNRs. The fast-light pulse is advanced by $t_{adv} = 27.4 \pm 1.1$ ns, which is 10.4% of its width. Using $t_{adv} = L/c - L/v_g$ with $L/c = 1.3$ ns, the group velocity can be found to be $v_g/c = -0.051 \pm 0.002$. Careful inspection of the fast-light pulse reveals that it has been compressed to a pulse width of 258 ns (1.9% compression) due primarily to the frequency dependence of the gain.

4.2 A gain-based slow-light medium

The system I use for the slow-light experiments is based on a single gain line, as discussed in Sec. 2.3. While this technique usually produces less extreme group velocities than other techniques such as EIT [8], it is extremely simple to set up and use. Like the fast-light system described in the previous section, this slow-light system is based on Raman gain.
Figure 4.6: Gaussian pulses after propagating through the fast-light setup (dashed line) and through vacuum (solid line). The pulses shown here are averages of 50 individual pulses and have been smoothed. The fast-light pulses propagate with $v_g/c = -0.051 \pm 0.002$ and are advanced by $t_{adv} = 27.4 \pm 1.1$ ns.

4.2.1 Experimental slow-light system

As shown in Fig. 4.7, the setup used for the slow-light experiments is almost identical to the fast-light setup. The physical apparatus differs only in that a single pump and cell are used. In practice, this amounts to simply blocking one of the pumps.

For the slow-light experiments, the cell is heated to 130° C, creating an atomic number density of $3.5 \times 10^{12}$ cm$^{-3}$. The pump frequency $\omega_d$ is set to 2.61 GHz to the high-frequency side of the $D_1$, which is 2.87 GHz higher than the lowest-frequency ($F = 2 \rightarrow F' = 1$) transition. The probe is tuned to the center of the single resulting gain resonance at $\omega_0 = \omega_d + 462$ MHz. These frequencies are shown in Fig. 4.8.

Figure 4.9a shows the experimentally measured gain profile (dots) for the slow-light system. Also shown is a fit of this profile to a Lorentzian. Panels b and c of Fig. 4.9 show the pulse spectra corresponding to the Gaussian pulses and
Figure 4.7: Experimental setup for the slow-light experiments.
Figure 4.8: Level diagram for the slow-light experiments.
**Figure 4.9:** Panel (a) shows the experimentally measured gain profile for the slow-light system (dots) and a calculated fit to a Lorentzian (line). Panel (b) shows the power spectra of the Gaussian pulse (solid), “0” symbol (dashed) and “1” symbol (dotted).
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to the pulse symbols (described further in the following chapters). These spectra are normalized to unit height for convenient comparison and correspond to the smoothed pulses described in Sec. 4.3. These spectra are calculated by computing the square magnitude of the Fourier-transformed electric field. Figure 4.10a shows the predicted group index \( \frac{c}{v_g} \) using the fit in Fig. 4.9a. In this case, the gain varies more rapidly as a function of frequency at the pulse frequency than in the fast-light case. As a result, one would expect more distortion, mostly in the form of pulse expansion, as described in Ch. 2.3.1.

4.2.2 Experimental slow-light observations

Figure 4.11 shows the result of propagating Gaussian pulses through this slow-light system. The 265-ns-long (FWHM) pulses propagate through both the slow-light medium and through vacuum. The peak power for the vacuum pulse is 24.4 \( \mu \text{W} \) and for the slow-light pulse is 24.0 \( \mu \text{W} \). However, because the injected slow-light pulse is weaker by a factor of \( 10^{0.5} = 3.16 \), this corresponds to pulse amplification by a factor of \( 3.1 = e^{1.13} \). These pulse powers are chosen according to the same criteria as the fast-light powers, as described in Sec. 4.1.4. The slow light pulses are delayed by \( t_{del} = 67.5 \pm 2 \text{ ns} \), corresponding to a relative delay of 25\%. Using \( L/c = 0.66 \text{ ns} \), one can infer that \( v_g/c = 0.0097 \pm 0.0003 \). In the slow-light case, the frequency-dependence of the gain leads to pulse expansion (by 17\%) to 311 ns.
Figure 4.10: Panel (a) shows the predicted group index \((c/v_g)\) using the fit in Fig. 4.9a. The horizontal line is drawn at \(c/v_g = 1\) and separates fast-light (below) and slow-light (above) regions. Panels (b) and (c) are identical to those in Fig. 4.9.
Figure 4.11: Gaussian pulses after propagating through the slow-light setup (dashed line) and through vacuum (solid line). The pulses shown here are averages of 50 individual pulses and have been smoothed. The slow-light pulses propagate with $v_g/c = 0.0097 \pm 0.0003$ and are delayed by $t_{del} = 67.5 \pm 2 \text{ ns}$.

4.3 Theoretical model of optical pulse propagation

In order to better understand and analyze the experimental systems described above, I present a numerical model of pulse propagation in fast- and slow-light media. This model is also used in the implementation of the measurement technique described in Ch. 5.

Fundamentally, the numerical model is a technique for numerically computing Eq. 2.9 from Sec. 2.1.1, repeated here for convenience.

$$A(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(0, \omega - \omega_0) e^{i(k(\omega)z - \omega t)} d\omega$$  \hspace{1cm} (4.2)

The propagation of an arbitrary input pulse $A(0, t)$ through an arbitrary medium can be computed in two steps. The first step is to calculate the Fourier transform of
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the input pulse shape to get $A(0, \omega - \omega_0)$. The second step is to perform the inverse Fourier transform of the quantity $A(0, \omega - \omega_0) e^{ik(\omega)L}$, yielding the pulse amplitude $A(L, t)$ as a function of time at depth $L$ in the medium.

When possible, these two stages should be computed analytically. For example, if the pulse is a Gaussian and $k(\omega)$ is purely quadratic, the entire computation can be performed analytically, as shown in Sec. 2.1.3. However, if the function $k(\omega)$ is not so simple, the inverse transform may not be analytically tractable. In that case, it can be performed via the fast fourier transform (FFT) [29] or another suitable algorithm.

In the next two sections, I discuss two important considerations when implementing this model; the causal properties of algorithms and modeling supporting experimental apparatus. Finally, I describe the complete implementation of the model and present several examples of its use.

4.3.1 Causal and acausal algorithms

When creating numerical models of systems that are sensitive to the propagation of information, it is often important to be aware of the causal properties of the chosen algorithms. An algorithm is causal if it does not allow information from one part of the input to “leak” into an earlier part of the output. For example, imagine an electronic signal passing through an electronic filter. The behavior of the input after time $t = 0$ should not affect the behavior of the output before time $t = 0$. Such a device would be acausal. Similarly, a numerical model used to simulate such a filter should obey the same rule.

It is, of course, possible to design a mathematical process that is deliberately acausal. In fact, it is often quite useful when digitally filtering a signal. The
standard “running average,” where each data point is replaced with the average of itself and its $N$ nearest neighbors is an example of an acausal filter. There is no physical device that can reproduce this effect because the output at any given time depends on input that has not yet arrived. However, the desired smoothing effects can often be achieved without the acausal behavior. For example, if each point is instead replaced with the average of itself and its $N$ preceding neighbors, the result will be identical except for a shift in the time axis. Examples of these two algorithms are shown in Fig. 4.12. For many applications, these two algorithms are equivalent. For other applications, one or the other is preferred. For example, the first algorithm (shown in Fig. 4.12a) minimizes the mean temporal shift between a broad feature in the input and the corresponding feature in the output; a smooth symmetric pulse peak will remain unshifted in time. However, this comes at the cost of causality; the pulse front will be shifted to earlier times, as shown in Fig. 4.12b.

A slightly more insidious form of acausal “leakage” comes from certain numerical algorithms that are sensitive to floating point or “roundoff” errors. For example, consider the discrete Fourier transform (DFT). For simplicity, let us consider the matrix implementation of the DFT [29], where the transform operator and its inverse are $N \times N$ matrices of the form

\begin{align}
F_{pq} &= \frac{1}{\sqrt{N}} e^{i2\pi pq/N}, \\
F^{-1}_{pq} &= \frac{1}{\sqrt{N}} e^{-i2\pi pq/N}. 
\end{align}

(4.3)

(4.4)

For a given time series $s$ of length $N$, it is true that $F^{-1}F s = s$ because $F^{-1}F = I$, where $I$ is the identity matrix. However, when the matrix multiplication is
Figure 4.12: Both panels show simple running averages. Each point in the output is an average of three adjacent points in the input, as shown by the boxed points. The algorithm used to generate the data in (a) is acausal, leading to early detection of the rise in the signal. The algorithm in (b) is causal. Note that in this example, the two outputs only differ by a temporal shift of one timestep.
performed numerically with roundoff error, it is possible—indeed, likely—that the product $\mathcal{F}^{-1}\mathcal{F} \neq I$. Typically, the off-diagonal elements will be small, on the order of the floating point precision, but non-zero. Thus, not only is the numerically-computed $\mathcal{F}^{-1}\mathcal{F}$ different from the original $s$, but any value in $\mathcal{F}^{-1}\mathcal{F}s$ can be affected by any other value in $s$ regardless of their temporal relationship because every element in $\mathcal{F}^{-1}\mathcal{F}$ can be non-zero. Therefore, changing some value that comes late in the time series $s$ may result in an acausal change in early values of $\mathcal{F}^{-1}\mathcal{F}s$ [57].

Obviously, the example provided above poses no practical limitation; there are better ways to generate an identity matrix. However, similar acausal behavior may be relevant if one needs to transform into frequency space, perform some operation, and then transform back. As described above, the numerical solution of Eq. 4.2 proceeds in precisely this fashion; the transforms of two quantities are multiplied and then the product is inverse-transformed. However, just because the algorithm is subject to this floating-point-acausal behavior, that does not mean it is useless. In practice, decisions about whether possible acausal behavior is problematic must be made on a case-by-case basis. For the present experiments, the magnitude of the floating point error is many orders of magnitude smaller than the noise that is deliberately introduced to mimic the modeled experiment. Therefore, any tiny acausal behavior from the algorithm is dominated by noise and therefore undetectable.

### 4.3.2 Modeling finite-response-time equipment

One of the biggest challenges in modeling a complex experiment such as this is modeling the equipment used to perform the experiment. Each of the devices used to generate and detect the pulse shapes has a finite response time and so effectively
filters the pulse. Often the precise filter characteristics can be very complicated.

In the experiments described herein, there are three major components that lead to this filter behavior; the arbitrary waveform generator (AWG, Stanford DS345), the acousto-optic modulator (AO) and driver, and the detector (New Focus 1801-FS-AC). The AWG and detector primarily acquire their filter behavior from simple filtering electronics. These electronic filters are part of the manufacturer’s design, and so they are well documented. The AWG includes a 7-pole low-pass Bessel filter with a 10 MHz cutoff frequency. A circuit diagram for the detector is not available, but it is specified to have a 3 dB response range of 25 kHz to 125 MHz.

Unfortunately, the filter characteristics of the AO and driver are difficult to characterize. They depend on the electronics within the driver, the electronics within the AO, the propagation of the acoustic field within the crystal, and the interaction of the acoustic field with the transverse profile of the optical beam. To make matters worse, the pulse AO is double-passed, requiring that the return beam be precisely overlapped with the first-pass beam. Because the speed of sound in the crystal is relatively slow, misalignment of only a few μm can lead to significant changes in the response characteristics of the AO. Also, the optical diffraction efficiency is a complicated, unknown (but very nonlinear) function of the input voltage, further complicated by the double-pass setup.

The physical apparatus is modeled using two digital filters, a 7-pole low-pass Bessel filter with a 10 MHz cutoff frequency (as is specified for the AWG) and a single-pole band-pass Butterworth filter with cutoffs at 25 kHz and 125 MHz. Various attempts to find filter characteristics of the AO that improved the similarity between model and experiment have been largely unsuccessful. Therefore, no dedicated AO filter is used. The filters are implemented using an iterative algorithm [58]
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Figure 4.13: Logical flow diagram of the numerical simulations. The pre-filter and post-filter stages correspond to the effects of the pulse creation and detection apparatus respectively.

that is equivalent to convolving the input with an impulse response function. This approach is computationally efficient and is not susceptible to the causality pitfalls described in the previous section.

4.3.3 Applying the exact numerical model

The techniques described in the previous sections can be combined to model the behavior of the entire system and the resulting detected pulses. This process is described graphically in Fig. 4.13. First, the exact input pulses are created. Next, the pulses are pre-filtered to mimic the pulse creation apparatus. These pulses are then Fourier transformed, and the pulse transform is multiplied by \( e^{ik(\omega)z} \), where \( k(\omega) \) is based on the measured transmission properties of the medium. Next, white Gaussian noise is added to the pulses such that the standard deviation of the final modeled pulses matches that of the experimental pulses. Finally, the pulses are post-filtered to mimic the detector. In principle, the order of these operations is largely irrelevant because they are all linear and the noise is additive.

Figures 4.14, 4.15, and 4.16 show comparisons between measured experimental pulses and numerically modeled pulses for the fast, slow, and vacuum cases respectively. The vacuum pulses are those used in the slow-light experiment. Note
that the same technique was used in all cases, but the implementation of the tech-
nique was refined after the fast-light experiments. As a result, the noise and pulse-separation-time is better-matched in the slow-light example. Neither of these issues significantly impacts the results; as will be discussed in the next chapter, it is only important that the difference between pulses in the vacuum and the medium is similar for theory and experiment. That is, theoretical and experimental pulses are never directly compared—only the effects of propagation through the medium are compared.

The numerical model developed in this section serves two practical purposes. The first is to provide physical insight and to help build intuition about the behavior of the physical system. It is also useful as a quick test-bed for new experimental ideas. The other purpose is as part of the analysis of the experimental data, and the measurement of the information velocity, as described in the next chapter.
Figure 4.14: Comparison of experimental and modeled fast-light pulses. Each of these is a single pulse with no data processing except for scaling to unit peak height (for the smooth Gaussian).
Figure 4.15: Comparison of experimental and modeled slow-light pulses. Each of these is a single pulse with no data processing except for scaling to unit peak height (for the smooth Gaussian).
Figure 4.16: Comparison of experimental and modeled vacuum pulses. These are the vacuum pulses used in the slow-light experiments. Each of these is a single pulse with no data processing except for scaling to unit peak height (for the smooth Gaussian).
Chapter 5

Measuring the information velocity

In the previous chapters, I have discussed the physics of fast- and slow-light pulses, and described the creation of media capable of transmitting such pulses. I have also described the current state of research on the velocity of information and on fast- and slow-light pulses. In this chapter, I will describe a new technique for measuring the information velocity. This technique makes it possible for the first time to truly measure the velocity of information on optical pulses. This fundamentally new approach is based on information-theoretic principles rather than the measurement of some arbitrary mathematical or physical feature. By sending distinct symbols through a medium and measuring the time when those symbols can be identified, it is possible to directly measure the arrival of information. This technique consists of two major parts; quantifying detected information as a function of time via the bit error rate (BER), and using this BER measurement to calculate the information velocity.

Although this technique will be employed in the next chapter for the specific cases of the fast- and slow-light media described in Ch. 4, it is presented here in its general form. The reason for this generality is because this technique is very powerful and can be used to measure the information velocity in a wide variety of
media, and for any type of pulses.

## 5.1 Quantifying detected information

The standard way to measure any velocity is to use the simple formula

\[
v = \frac{\delta x}{\delta t},
\]

(5.1)

where \(\delta x\) is the distance the object in question travels, and \(\delta t\) is the time it takes to travel that distance. In the case of solid objects, both of these are easy to measure because the position of such objects (classically, at least) is well defined. For more complicated quantities, it is often less clear. Describing the spacetime location of information is particularly complicated because information is not associated with any physical object in a simple way. Information can be encoded on matter or energy, but the relationship between the information and its carrier varies with encoding scheme, and often changes as the information propagates.

In the following sections, I describe a technique for quantifying the detected information in terms of the bit error rate (BER). For an arriving pulse, this quantity varies as a function of time. Using this quantity, it is then possible to assign a time at which information arrives at a specified position.

### 5.1.1 Creating distinct symbols

As described in Sec. 1.3.2, the transmission of information requires the creation and transmission of distinct symbols. The receiver, by detecting and identifying the transmitted symbols, can reconstruct the information that the sender intended to communicate. The specific number and optical characteristics of the symbols are
largely arbitrary, but there must be at least two symbols. Note that there are many ways the symbols can differ. For example, two optical pulse symbols can have identical shapes, but different polarizations. Or two otherwise identical symbols could arrive shortly before and shortly after a regular time step. The variations are endless. The technique for measuring $v_i$ described in this chapter can be applied to any of these “alphabet” choices, although some choices might produce more accurate or easily interpreted results, as described below.

In practical communication systems, there are a number of important considerations when choosing an alphabet: the polarization-preserving or nonlinear properties of the medium, bandwidth limitations, etc. For experiments to measure the information velocity, the most important consideration is when the symbols can be identified. The information is encoded on a symbol when the sender must commit to that symbol, at the moment when the symbols first differ. Before that moment, the sender has sent no information, and before that point arrives at the receiver, the receiver can detect none. If the symbols remain similar long after the moment they first differ, then it will be difficult for the receiver to identify the symbols.

An example of an alphabet composed of symbols that are difficult to distinguish when they first begin to differ is shown in Fig. 5.1, where the two symbols are near-Gaussian pulses with peaks at different times. The symbol intensities are given by the formula

$$I_j(t) = I_0 e^{-(t-(t_{C_j}+t_{C_0})))^2/t_f^2} \Theta(t),$$

(5.2)

Where $j$ is either 0 or 1. Here, $t_f$ is the Gaussian half width at $1/e$ intensity, $\Theta(t)$ is the Heaviside step function, $t_{C_j}$ is the center of the Gaussian function for pulse $j = 0$ and $t_{C_j} + t'_{C_j}$ is the center for pulse $j = 1$. For the pulses shown in Fig. 5.1, $t_{C} = 250$ ns, $t'_{C} = 50$ ns, $I_0 = 1$, and $t_f = 100$ ns. The symbols are not true
Gaussian pulses because they have zero intensity before time \( t = 0 \) ns. As a result, the symbols first differ at time \( t = 0 \) ns. That is the time when the information is encoded and (assuming no propagation delay) when it can first be detected.

As this alphabet is shown in Fig. 5.1a, the information can still be detected immediately after time \( t = 0 \); one need only have high enough resolution to distinguish the intensities, as shown in Fig. 5.1b. However, in the presence of noise, the symbols cannot be reliably identified until much later when the difference between the symbols becomes comparable to the noise, as seen in Fig. 5.1c. This delay is called the detection latency, and will be discussed in more detail in Sec. 5.2. This issue of noise and the detection latency is an important consideration when choosing a symbol alphabet.

The alphabet used in the present experiments is shown in Fig. 5.2. For this alphabet, the symbols are identical until time \( t = 0 \) ns, when they diverge rapidly. As a result, it is much easier to identify the pulses for small positive times \( t \). Even in the presence of moderate noise, these symbols can be quickly identified. For example, Fig. 5.2b shows the same symbols with the same amount of noise added that was added to the pulses in Fig. 5.1c, and yet the symbols can be identified immediately after time \( t = 0 \). However, Fig. 5.2 shows the pulses in their ideal form; in the experiment, the equipment used to generate and detect the pulses affects their shape and makes them more difficult to distinguish. The effects of the apparatus are discussed in Sec. 4.3.2 and examples of the resulting pulses are shown in Fig. 4.16a.
Figure 5.1: An example of an alphabet with symbols that are difficult to distinguish until long after the information is encoded. The symbols are similar for a long time after they first differ. Panel (a) shows the entire symbols. Panel (b) shows a zoom of panel (a). Panel (c) shows the same zoom, but with noise added so that the symbols are no longer identifiable immediately after time \( t = 0 \).
Figure 5.2: The two symbols that make up the alphabet used in the experiments to measure $v_1$. Note that the symbols are identical until they diverge rapidly. They are therefore easy to distinguish immediately after the information is encoded. Panel (a) shows the ideal noise-free symbols, and panel (b) shows the symbols with moderate noise added. Even with noise, these symbols can be immediately distinguished.
CHAPTER 5. MEASURING THE INFORMATION VELOCITY

5.1.2 Detecting information: matched filters

Once the symbol alphabet is chosen, the symbols can be transmitted via the communication system in order to transmit information; the sender can pick a symbol (“0” or “1,” for example) and send it to the receiver. However, the receiver must have a way of identifying the symbol. When designing a communication system, careful thought must go into this identification process. In general, it is beneficial to use an identification scheme that correctly identifies as many incoming symbols as possible. For measuring the information velocity, the chosen identification scheme must also be able to perform the identification at any requested time.

An identification scheme can never identify all symbols correctly because there will always be noise. This noise can cause one symbol type to look like another type, and be (mis-)identified accordingly. Given that it is impossible for the identification scheme to perfectly identify the symbols, it is clearly unreasonable to expect it to be perfect at any arbitrary time. However, the scheme should correctly identify as many pulses as possible. In general, there are many possible identification schemes, but the one that most successfully identifies incoming pulses should be used. Correct and early identification is important because identifying the incoming symbols later than they could be identified will lead to erroneous arrival times and incorrect $v_i$ measurement.

The identification scheme used to identify incoming pulses for the experiments described in this thesis is based on an integrate-and-dump matched filter [59], wherein each incoming pulse is compared to two\footnote{The scheme described here is designed for a binary symbol alphabet, but can be extended to larger alphabets.} references waveforms $R_0(t)$ and $R_1(t)$. These waveforms are chosen to be the ideal pulse shapes after traveling
through the system. Therefore, they should include any repeatable pulse distortion that the system introduces. One way to generate $R_j(t)$ is to transmit many pulses of type $j$ through the system and then average them. This averaging removes noise, but preserves any repeatable distortion introduced by the system.

The incoming pulse, called the test pulse waveform and denoted by $x(t)$, is compared to each of the references using the similarity integral

$$S_j(\tau) = \frac{1}{\alpha_j(\tau_a) N_j(\tau)} \int_{t_s}^{\tau} x(t) R_j(t) dt \quad (j = 0, 1),$$

which is numerically large when $x(t)$ and $R_j(t)$ are similar and small when they are dissimilar. Here, $t_s$ is the integration start time chosen to be long before symbols diverge. In the present experiments, $t_s$ corresponds to the first data point in the time series, which is at $t \sim -500$ ns. The comparison only includes the part of $x(t)$ before the integration stop time $\tau$. Successful comparisons are normalized to $S_j(\tau) \sim 1$ by including the normalization integral

$$N_j(\tau) = \int_{t_s}^{\tau} R_j^2(t) dt.$$  

The normalization constant

$$\alpha_j(\tau_a) = \frac{1}{N_j(\tau_a)} \int_{t_s}^{\tau_a} x^2(t) dt,$$

can be used to compensate for intensity fluctuations between test waveforms. For example, if there are slow variations in the gain or attenuation in the system that cause some pulses to be larger or smaller than others, while retaining their general shape. Here, $\tau_a$ is a time chosen arbitrarily before the symbols diverge, but after
they have reached an intensity that is large compared to the noise.\(^2\)

The results of comparing the test waveform to the two reference waveforms can be combined by subtracting the similarity integrals to produce the difference integral

\[
D(\tau) = S_0(\tau) - S_1(\tau). 
\] (5.6)

If \(x(t)\) is similar to \(R_0(t)\) for \(t \leq \tau\), then \(D(\tau) \sim 1\). On the other hand, if \(x(t)\) is instead similar to \(R_1(t)\), then \(D(\tau) \sim -1\). It is also possible that the test pulse waveform cannot be clearly identified, as is the case before the symbols diverge. In that event, \(D(\tau) \sim 0\).

With the difference integral \(D(\tau)\) calculated, the test waveform can be identified (although not necessarily correctly identified) by comparing the waveform’s difference integral \(D(\tau)\) to some threshold value \(d(\tau)\), which may vary as a function of \(\tau\). When \(D(\tau) > d(\tau)\), the test waveform is considered a “0,” and when \(D(\tau) < d(\tau)\), the test waveform is considered a “1.” In a perfectly symmetric case, the threshold value \(d(\tau)\) would be precisely zero for all \(\tau\), but that is not the case in general.

Using this matched filter identification scheme, it is possible to identify received pulses at any moment in time. Most important, an identification of a waveform \(x(t)\) performed at time \(\tau\) uses only the part of \(x(t)\) for \(t \leq \tau\). Therefore, with sufficiently fast equipment, this identification scheme can be implemented in real-time.

### 5.1.3 Bit error rate

The threshold value \(d(\tau)\) described in the previous section should be chosen to minimize the fraction of pulses that are incorrectly identified. This value is quantified

\(^2\)If a suitable \(\tau_a\) cannot be chosen before the symbols diverge, as for the symbols in Fig. 5.1, then one must simply use \(\alpha_j = 1\).
by the bit error rate (BER), which is simply the fraction of symbols (for a binary
or two-symbol alphabet) that are misidentified [59]. In principle, the BER can take
on any value between 0 and 1, but in practice, only values between 0 and 1/2 are
relevant. For example, if a chosen identification scheme misidentifies 3/4 of the
incoming symbols, then the BER is 0.75. However, one can trivially convert this
to 0.25 by simply reversing all identifications. The BER, a measure of detected
information, can be used to choose the threshold $d(\tau)$; choose $d(\tau)$ to result in the
best (smallest) BER.

There are two common ways to calculate the BER, and thereby choose the
threshold $d(\tau)$, both of which I will describe here. Although I only use the second
approach in my experiments, I present the first technique here for two reasons.
First, it is much easier to understand and visualize. Second, it serves as a clear
stepping-stone to the second approach, which solves some of problems with the first
approach.

Both of these BER calculation techniques require only the values of the differ-
ence integral $D(\tau)$ for the test pulses. Because the matched filter described in the
previous section so effectively distills the similarity of the test pulses to the reference
pulses, the precise shape of those pulses is largely irrelevant. Only the probability
distributions of the $D(\tau)$ integrals are important, and almost any symbol alphabet
will lead to qualitatively similar distributions of $D(\tau)$. To demonstrate these tech-
niques below, I use randomly-generated Gaussian distributions of $D(\tau)$ centered
about $D(\tau) = 1$ for type-0 symbols (that is, those corresponding to a binary “0”)
and $D(\tau) = -1$ for type-1 symbols. The widths of these distributions are chosen to
demonstrate the distinguishability of the pulses.
Measured BER

The first approach to calculating the BER and choosing the threshold \( d(\tau) \) is both simple and obvious: collect many test pulses of known type, and then choose \( d(\tau) \) to minimize the measured BER. This approach is shown graphically in Fig. 5.3 for the example distribution described above. In this example, the distributions of \( D(\tau) \) for the two symbol types are clearly different, but still overlapping, suggesting that the measurement time \( \tau \) comes after the information was encoded, but while the symbols are still difficult to distinguish. Figure 5.3a shows the distributions of \( D(\tau) \) for both symbol types. As expected, symbols of type 0 have values of \( D(\tau) \approx 1 \) and symbols of type 1 have \( D(\tau) \approx -1 \). These values can be used to calculate a directly-measured BER simply calculating the number of pulses that would be misidentified for each threshold value \( d(\tau) \). For any threshold \( d(\tau) \), all “0” symbols with \( D(\tau) < d(\tau) \) would be misidentified, and any “1” symbols with \( D(\tau) > d(\tau) \) would be misidentified. Dividing the sum of these two quantities by the total number of symbols sent, the BER can be calculated as a function of \( d(\tau) \), as shown in Fig. 5.3b.

While this approach for calculating the BER and choosing \( d(\tau) \) is functional for poor BERs where there is some significant overlap between the two distributions of \( D(\tau) \), it becomes less useful as the two distributions become more separated. This breakdown in usefulness is shown in Fig. 5.4, which is similar to Fig. 5.3 except that the two distributions of \( D(\tau) \) are completely distinct. As a result, the measured BER is exactly zero for a large range of \( d(\tau) \) near \( d(\tau) = 0 \). It is reasonable to expect that the BER is not truly zero over this range—eventually, a pulse will fall on the wrong side of the threshold—but we do not have enough samples to exhibit this behavior. This under-sampling problem can be especially troublesome in real-
Figure 5.3: Simple method for choosing the identification threshold $d(\tau)$ using the BER. Panel (a) shows an example distribution of difference integrals $D(\tau)$ for 100 pulses of each type. In this example, $\tau$ is fixed and chosen so that the pulses can be distinguished, but not perfectly. Each circle represents a single pulse of known type. Its horizontal position indicates the value of $D(\tau)$ for that pulse. Histograms of these points are also shown. Panel (b) shows the measured BER as a function of threshold value $d(\tau)$. As one would expect, the lowest BER values are achieved for thresholds near $d(\tau) = 0$. 
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Figure 5.4: This figure is the same as Fig. 5.3 except that the measured distributions of $D(\tau)$ for the two symbol types are completely distinct. As a result, there is a range of $d(\tau)$ near $d(\tau) = 0$ where the measured BER is exactly zero.
world communication systems, where the BER can be on the order of $10^{-12}$ or smaller [59]. This problem can be addressed by using a variation on this technique, as described in the next section.

**Fit-based BER**

When the number of pulses is small and/or the distributions of $D(\tau)$ are very distinct, better BER results can be achieved by fitting the distributions of $D(\tau)$ to some appropriate function, and finding the BER analytically from those fits. Variations in $D(\tau)$ typically come from random processes like noise in the system, and so the distributions of $D(\tau)$ are often Gaussian in shape. It is rarely practical to prove that the distributions are Gaussian for real-world systems due to the enormous complexity, so the standard technique is simply to attempt a Gaussian fit and confirm that the distribution fits well to a Gaussian [59].

Gaussians have the convenient property that if the two distributions of $D(\tau)$ are normalized to area 1/2, then the BER is equal to the total area of each Gaussian that is on the “wrong” side of the threshold $d(\tau)$. This is represented mathematically in terms of the complementary error function as

$$\text{BER}(d) = \frac{1}{4} \left[ \text{erfc} \left( \frac{d - x_1}{\sigma_1} \right) + \text{erfc} \left( \frac{x_0 - d}{\sigma_0} \right) \right], \quad (5.7)$$

where $x_j$ are the centers and $\sigma_j$ the widths of the Gaussian distributions. Choosing $d$ to minimize this BER, one finds that the optimum threshold $d(\tau)$ is given by

$$d_0(\tau) = \frac{(x_0\sigma_1^2 - x_1\sigma_0^2) - \sigma_0\sigma_1\sqrt{(x_0 - x_1)^2 + (\sigma_1^2 - \sigma_0^2)\ln(\sigma_1/\sigma_0)}}{\sigma_1^2 - \sigma_0^2} \quad (5.8)$$

or $d_0(\tau) = (x_0 + x_1)/2$ when $\sigma_0 = \sigma_1$. This optimum threshold is the point where the
two Gaussian curves cross. This technique for choosing a threshold and calculating the BER is shown graphically in Fig. 5.5 using the same data used in Fig. 5.4. By comparing these two figures, one can see that this fit-based technique allows for a much better choice of $d_0(\tau)$ and a much better estimate of the BER for the same input data.

Using the technique described above, the detected information can be quantified as a function of detection time $\tau$ using the BER:

$$\text{BER}(\tau) = \text{BER}(d_0(\tau)).$$

(5.9)

When the BER is equal to $1/2$, no information is being detected, and as $\text{BER} \rightarrow 0$, the information is being detected perfectly. Therefore, the moment when the BER first drops below $1/2$ is the moment when information begins to arrive.

5.2 Determining the information velocity

In the previous section, I described a method for quantifying the detected information as a function of time using the BER. In this section, I will describe a process by which such measurements can be used to measure the information velocity $v_i$.

5.2.1 Deriving an expression for the information velocity

As outlined briefly in Sec. 1.3.3, we would like to define the information velocity $v_i$ according to

$$v_i \equiv \frac{L}{t_1 - t_0}.$$  

(5.10)
Figure 5.5: This figure is identical to Fig. 5.4 except the distributions of $D(\tau)$ have been fit to Gaussian profiles. The BER corresponding to these Gaussian fits is shown (dashed). Using this technique, it is possible to determine the ideal threshold ($d_0(\tau) \approx 0.046$) and BER at that threshold (BER $\approx 0.005$).
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where $L$ is the length of the medium, $t_0$ is the time when the information enters the medium, and $t_1$ is the time when the information leaves the medium. Unfortunately, as described below, these two times cannot be measured directly.

The time $t_0$ might seem straightforward to determine; it is completely controlled by the sender, after all. However, there are inevitably a number of delays introduced by various devices that must be characterized, often extremely precisely. These delays are difficult to characterize well because they both delay and smooth the pulses. This challenge can be overcome by using the same experimental system to send identical pulses through a length $L$ of vacuum and by assuming that the information velocity in vacuum $v_{i,vac} = c$. Solving Eq. 5.10 for $t_0,vac$ in the vacuum case, and substituting back in for the fast/slow light medium, we find

$$v_{i,med} = \frac{L}{t_{1,med} - (t_{1,vac} - L/c)}.$$  \hspace{1cm} (5.11)

This substitution removes the need to measure $t_0$ directly, and replaces that measurement with a second measurement of $t_1$, this one for vacuum pulses. This approach is quite logical—only the behavior of the medium is of interest, so sending the pulses through both vacuum and the medium allows for isolation of effects that are due to the medium.

The only remaining measurements are $t_{1,med}$ and $t_{1,vac}$. Recall that the time $t_1$ is the moment when information first leaves the medium (or vacuum). In terms of the BER, this is the moment when the BER first deviates from 1/2. However, the BER is only ever equal to 1/2 in the ideal case where the distributions of $D(\tau)$ have zero width, as shown in Eq. 5.7. As long as there is some noise that causes different pulses to produce different values of $D(\tau)$, then the Gaussian distributions of $D(\tau)$ will have some overlap, and $\text{BER}(\tau) < 1/2$. Therefore, one must instead set
a threshold BER and use the time that the measured BER crosses that threshold. This threshold crossing will occur at time $t_c$, which is later than $t_1$. I refer to this delay

$$\Delta t \equiv t_c - t_1$$

as the detection latency. It is important to choose a high threshold in order to minimize $\Delta t$, but higher thresholds result in larger uncertainty in the crossing time.

The detection latency, discussed in more detail in the next section, is shown qualitatively in Fig. 5.6. Panel (a) shows two symbol shapes generated using the model described in Sec. 4.3. The original pulses diverged discontinuously at time $t = 0$, but due to noise and the filtering effects of the modeled apparatus, the symbols cannot be distinguished until much later. Panel (b) shows a close-up of the region of interest. The time $t_1 = 0$ when the information exits the medium (if the pulses are sent through a medium with length $L = 0$) is marked. However, the pulses cannot yet be identified. Only at some later time $t_c$—chosen arbitrarily for this example—do the symbols separate. This delay $\Delta t \equiv t_c - t_1$ is the detection latency. For comparison, panel (c) shows the same data without noise. Obviously, the symbols can be identified earlier. With a sufficiently sensitive detector, they could in principle be identified immediately after time $t = 0$. However, for the parameters used here, the separation immediately after time $t = 0$ is less than the numerical precision of the calculation.

After choosing a suitable BER threshold to balance the issues of detection latency and uncertainty in the BER crossing time, one can then measure $t_c$ for both vacuum and the medium. Rewriting Eq. 5.11 in terms of their difference
Figure 5.6: Qualitative demonstration of the detection latency $\Delta t$. Panel (a) shows two symbol shapes generated using the model described in Sec. 4.3. The original pulses diverged discontinuously at time $t = 0$, but due to noise and the filtering effects of the modeled apparatus, the symbols cannot be distinguished until much later. Panel (b) shows a close-up of the region of interest. The time $t_1 = 0$ when the information exits the medium (for $L = 0$) is marked. However, the pulses cannot yet be identified. Only at some later time $t_c$—chosen arbitrarily for this example—do the symbols separate. For comparison, panel (c) shows the same data without noise, in which the symbols can be identified earlier.
$T_i \equiv (t_{c,\text{med}} - t_{c,\text{vac}})$ leads to

$$v_{i,\text{med}} = \frac{L}{T_i - (\Delta t_{\text{med}} - \Delta t_{\text{vac}}) + L/c}.$$  \hspace{1cm} (5.13)

If the detection latencies $\Delta t_{\text{med}}$ and $\Delta t_{\text{vac}}$ were identical, the detection latency would not be problematic; they would simply cancel each other in Eq. 5.13. Unfortunately, propagation through dispersive media can affect the waveforms in subtle ways. In addition to affecting the amount of noise on the pulses, the shape of the pulses is modified. For example, the narrow gain characteristics of both the fast- and slow-light media can smooth the symbol waveforms. As a result, the symbols that propagate through the fast- or slow-light media can separate more slowly than their vacuum counterparts. Even if they had the same amount of noise, the fast- and slow-light BERs would reach the target threshold later than the vacuum BER. As a result, the detection latency cannot be ignored. Furthermore, $(\Delta t_{\text{med}} - \Delta t_{\text{vac}})$ cannot be measured directly because it depends on the details of the noise and effects of propagation through the medium. However, it can be estimated using the theoretical model of the experimental system described in Sec. 4.3 such that

$$\Delta t_{\text{med},\text{th}} - \Delta t_{\text{vac},\text{th}} \approx \Delta t_{\text{med},\text{exp}} - \Delta t_{\text{vac},\text{exp}}.$$  \hspace{1cm} (5.14)

Consistent with previous models $[2, 11, 50]$, this model exhibits $v_{i,\text{med}} = c$. Substituting this and Eq. 5.14 into Eq. 5.13, we find

$$v_{i,\text{med}} = \frac{L}{T_{i,\text{exp}} - T_{i,\text{th}} + L/c}.$$  \hspace{1cm} (5.15)

Using this result, the velocity of information $v_i$ can be measured in a length $L$.
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of any medium using a total of four measurements of $t_c$: for the physical medium, for vacuum, for the modeled medium, and for the modeled vacuum.

It may seem strange to use a theoretical model to calculate what is advertised as an experimental result. The explanation is simply that the measured time difference $T_i \equiv (t_{c,med} - t_{c,vac})$ depends both on the information velocity in the medium and the detection latency difference between the medium and vacuum. With no further information, one cannot distinguish between these two effects. Therefore, the model is used to estimate the detection latency difference. Furthermore, it is only the modeled detection latency difference that impacts the final result.

5.2.2 Detection latency

For the most part, the derivation in the previous section is simple and straightforward. However, the physical interpretation of the detection latency $\Delta t$ bears further discussion. Most of the concepts presented and applied in this thesis are not fundamentally new. I have simply taken them from various disciplines (information theory, classical optics, etc.) and combined them in a new fashion. However, I am not aware of any previous work that addresses the existence of the detection latency.

Simply put, the detection latency is the delay between the time when information first arrives at the detector ($t_1$, as defined above) and the time one achieves some arbitrary degree of confidence that information has been detected ($t_c$). The detection latency is a property of the entire system, including the creation, transmission, and detection stages. It cannot be completely avoided and it is present for any medium, including vacuum. Even if there is no distance between the source and the detector, noise and finite response in the apparatus will render it impossible to detect the information at the same instant it is encoded.
The medium does affect the detection latency, though. If a medium reduces the signal-to-noise ratio (SNR) of the waveform, it will take longer to reach the target confidence (BER threshold) regardless of the velocity of information in that medium. A medium can also change the shapes of the pulses in such a way as to increase or decrease the detection latency. For example, in the noise-free numerical models of both the fast- and slow-light media, the separation of the symbols is slowed, even though there is no change in the time when the symbols first begin to separate, as shown in Fig. 5.7. This phenomenon, combined with finite noise, delays detection of the pulses that traveled through the medium relative to the vacuum pulses. If both sets have the same amount of noise, the vacuum pulses can be identified earlier because they separate more quickly.

It bears repeating that the detection latency arises from the combination of noise and symbol distortion. Neither of these alone introduces a detection latency. For example, Fig. 5.2 shows symbols that are undistorted but noisy. Because they are undistorted, they diverge rapidly so that their separation becomes larger than the noise immediately. In contrast, Fig. 5.6c shows symbols that are distorted, but noise-free. In this case, the symbols can still be identified with a sufficiently sensitive detector. The detection latency is therefore truly a combined effect of noise and distortion.

It is the detection latency that has perhaps led researchers to believe that information travels faster than \( c \) in fast-light media. Consider, for example, the simple case of smooth Gaussian pulses propagating with \( v_g \) faster than \( c \). If one simply measures the time when the pulses first reach their half-height for vacuum and the fast-light medium, one would conclude that information has traveled faster in the fast-light medium. However, in choosing this detection scheme, one has introduced
**Figure 5.7**: Demonstration of how a medium can introduce detection latency by changing the pulse shape. (a) shows vacuum (solid) and fast (dashed) pulses as predicted by the numerical model. In this case, no noise has been added. (b) shows the intensity difference between symbols at the time when the symbols begin to separate. Due to the effects of the medium, the fast-light symbols separate more slowly than the vacuum symbols, even though both types begin to separate at the same moment (found to be $L/c = 1.3\text{ ns}$ if one zooms in enough).
an artificially large detection latency. More importantly, the detection latency is not the same for vacuum and the fast-light medium. Ignoring these facts leads one to erroneously conclude that the information velocity is faster than $c$.

To properly consider smooth Gaussian pulses, one must carefully consider the detection latency. Because the information in encoded when the Gaussian is first turned on (or when the decision is made to not turn it on), the front is the feature that should be detected. In the case of a smooth Gaussian that is turned on very early, the discontinuous front may be obscured by noise [17, 51]. In that event, one must detect later, introducing a detection latency. As described in the previous section, the detection latency can be accounted for trivially if it is the same for the medium and vacuum, $\Delta t_{med} = \Delta t_{vac}$. However, when these quantities are not equal, the difference between them $\Delta t_{med} - \Delta t_{vac}$ can often be minimized by minimizing $\Delta t_{med}$ and $\Delta t_{vac}$ themselves. It is therefore advantageous to detect as close as possible to the point where the information is encoded.

Any information velocity measurement must consider the detection latency. It can never be eliminated entirely, and so it must be accounted for. Failing to do so amounts to a tacit assumption that $\Delta t_{med} = \Delta t_{vac}$, which is often invalid. As a result, according to Eq. 5.13, the measured information velocity can be grossly incorrect.

5.3 Summary

In this chapter, I have described a fundamentally new approach to measuring the information velocity $v_1$. By bringing modern communications techniques like the matched filter and BER to bear on the problem, I have devised a way to quantify the detected information as a function of time for optical pulses. Using the measured
information-arrival time, I have also derived a mathematical expression for the measured information velocity that incorporates the real-world effects of noise and pulse distortion. As I will demonstrate in the next chapter, this technique allows for the first direct measurements of the information velocity on optical pulses.
Chapter 6

The information velocity in fast- and slow-light optical pulses

In the preceding chapters, I have described techniques for creating fast- and slow-light media, and for measuring the velocity of information encoded on optical pulses. The characteristics of the media described in Ch. 4 are important accomplishments for the following reasons. The fast-light system, in particular, exhibits undistorted relative advancement that is at the forefront of current research, and was unheard-of only a few years ago. In Ch. 5, I described the information velocity measurement technique that I developed, which is the first approach that truly measures the velocity of information encoded on optical pulses. This is a new and important result that represents a major accomplishment in the field of information velocity research. However, all of these results, representing years of study, are tools created to perform exactly one task: to measure the velocity of information on fast- and slow-light optical pulses.

In this chapter, I will describe the experiments that I designed and conducted to measure the information velocity in the fast- and slow-light media. I will also present the results of these experiments, which are the culmination of this thesis. These results conclusively show that the velocity of information does not exceed $c$, as required by relativistic causality. These results are also the first test of Chiao
and coworkers’ proposal that the velocity of information is governed by \( v_i = c \) [11].

### 6.1 General techniques

There are a number of experimental choices and techniques that are identical for both the fast- and slow-light \( v_i \) measurements. This is possible because the experimental setup for the fast- and slow-light media described in Ch. 4 are so similar, and because the same measurement technique (described in Ch. 5) is used in both cases. Many of these choices are simply specific implementations of the general \( v_i \)-measurement technique described in the previous chapter.

#### 6.1.1 Symbol alphabet

One important choice when performing measurements of the information velocity is the symbol alphabet. The choice of symbol alphabet has been discussed generally in Secs. 5.1.1 and 5.2.2. In those sections, I presented the symbols used in these experiments as examples. The two symbols are Gaussian pulses with discontinuities near their peaks. Both symbols have identical Gaussian leading edges, but differ near the peak where one symbol ("0") rapidly drops to zero intensity, and the other ("1") jumps to become a Gaussian with twice its initial peak intensity. This can be written mathematically as

\[
I_j(t) = I_0 e^{-(t-t_C)^2/t_f^2}[1 + (2j - 1)\Theta(t)],
\]

where \( \Theta(t) \) is the Heaviside step function, \( j \) is the symbol type (0 or 1), \( t_C \) is the center of the Gaussian function, and \( t_f \) is the Gaussian half width at 1/e intensity. The information is encoded at time \( t = 0 \), when a choice must be made of which
Figure 6.1: The ideal symbols used for the measurement of the information velocity $v_i$. The information is encoded at time $t = 0$, where the symbols first differ.

shape to send. For times $t < 0$, the two symbols are identical. These shapes are shown in Fig. 6.1.

In practice, the symbols are not true Gaussian shapes for all times $t < 0$; there is some very early time ($t \ll -1 \mu s$) when the symbols are first turned on. However, at those times, the ideal pulse amplitude is much smaller than either the noise or the arbitrary waveform generator (AWG) resolution. Finally, the discontinuous turn-on is identical for both symbols. For all of these reasons, the pulse deviation from true Gaussian is irrelevant for the present experiments.

6.1.2 BER calculation

In both the fast- and slow-light cases, the process described in Ch. 5 is used to determine $v_i$. In each case, a total of 200 experimental pulses are transmitted;
100 through the (fast or slow) medium, and 100 through vacuum. Another 200 theoretical pulses are generated to mimic the experimental ones as described in Sec. 4.3.

The number of pulses is limited by a purely practical consideration; the need to acquire all pulses in a short amount of time. If a “set” of pulses takes too long to acquire, then the experimental apparatus can change significantly over the course of acquisition. Specifically, the frequency of the laser (Coherent 899) drifts by as much as several MHz over a few minutes. This drift can affect dramatic changes in the dispersive properties of the medium because the Raman process is sensitive to detuning, as shown in Eq. 4.1. The pulses can be acquired quickly by automating the process so that each pulse is automatically sent, detected and recorded, but this cycle still takes several seconds. The limiting delay is simply that the data is acquired by a fast oscilloscope (Tektronix TDS 680B) and must be stored. Either storing the data on a floppy disk or transmitting the data over GPIB to a computer takes several seconds per pulse.

Once acquired, each of the pulses plays two roles in the BER calculation. First, each pulse is used as a test waveform, as described in Sec. 5.1.2. Each pulse is also used to form one of the reference waveforms for every other pulse. Consider, for example, the 100 fast-light pulses: 50 of type-0 and 50 of type-1. Let us refer to these as $x_{0,p}(t)$ and $x_{1,p}(t)$ where $p = 0, 1, \ldots 49$. Each pulse $x_{i,p}$ is compared to two references $R_{j,p}(t)$ where both $i$ and $j$ are either 0 or 1. These references are constructed according to

$$R_{j,p}(t) = \frac{1}{N-1}\sum_{k=0}^{N-1}(1-\delta_{k,p})x_{j,k}(t),$$

(6.2)

where $N = 50$ is the number of pulses and $\delta_{k,p}$ is the Kronecker delta function. That
is, the reference $R_{j;p}(t)$ of type $j$ used for pulse number $p$ is an exclusive average of all\(^1\) pulses of type $j$ except number $p$.

It is absolutely essential that a pulse not be used as part of its own reference. That is, $R_{j;p}(t)$ must not be constructed using $x_{j;k}$ where $k = p$. The pulse $x_{j;k}$ could only be used to construct one of its two references (a pulse of type-0 cannot be used to construct a reference of type-1). Therefore, when compared to its two references, the pulse’s similarity to the correct reference would be artificially increased; noise on the pulse would match the noise in the reference that came from that very pulse. Even diluted by a factor of $N = 50$, using the naive construction scheme

$$R_{j;p}(t) = \frac{1}{N} \sum_{k=0}^{N-1} x_{j,k}(t) \quad (6.3)$$

dramatically skews the identification.

This identification skew appears as an improvement in the BER that is most visible when symbols should not yet be identifiable, when the BER should be approximately $1/2$. This effect can be easily observed by taking a set of $2N$ noisy pulses of all the same shape and randomly dividing them into two groups, calling these groups type-0 and type-1. If one then calculates the BER using the first (correct) reference scheme, the pulses cannot be identified, as one would expect. However, using the the second (incorrect) reference scheme, the pulses can be successfully identified. Using the 50 real vacuum pulses (as shown below in Fig. 6.2a) divided into two groups of $N = 25$, I find that the pulses can be correctly identified with their correct group with a BER $\approx 0.1$ using the naive approach, and the

\(^1\)Obviously, this scheme cannot be implemented in real time; the first pulses sent must be compared to references made from later pulses. In a practical communications system, the references would be generated first and so incoming pulses could be compared immediately. The scheme used here is chosen to make the most efficient use of the limited amount of data.
expected BER ≈ 0.5 using the correct approach.

6.2 The information velocity in fast-light optical pulses

Having fully described the experimental system and techniques, I can now present the primary results of this thesis: the information velocity in a fast-light optical medium. In this section, I will describe the results of my fast-light experiments in which I find that the information travels at subluminal speeds even though the group velocity vastly exceeds the speed of light in vacuum.

Figure 6.2 shows examples of single pulses after propagating through (a) vacuum, (b) the fast-light medium, (c) numerically modeled vacuum, and (d) the numerically modeled fast-light medium. Recall from Sec. 5.2.1 that the slight difference in noise between the experiment and model is not critical because only the detection latency difference for each case is relevant. For example, adding noise may affect the detection latencies $\Delta t_{med}$ and $\Delta t_{vac}$, but it does not significantly impact the difference $\Delta t_{med} - \Delta t_{vac}$. The time origins of the experimental and modeled pulses cannot be directly compared because the origin is arbitrarily chosen for the experimental pulses. For the modeled pulses, time $t = 0$ corresponds to the time information was encoded. The corresponding time is not known for the experimental pulses because the electronic delays in the apparatus are extremely difficult to characterize with the necessary precision. The pulse powers are the same as described in Sec. 4.1.4, with the later half of the pulses either increasing or decreasing in intensity.

Due to the noise present on the pulses in Fig. 6.2, it is difficult to see the effects of propagation through the fast light medium. However, by averaging the 50 pulses
Figure 6.2: Examples of single pulses after propagating through (a) vacuum, (b) the fast-light medium, (c) numerically modeled vacuum, and (d) the numerically modeled fast-light medium.
of each type, as shown in Fig. 6.3, we see that the effects are striking. All four types of experimental pulses are shown, and two facts are immediately clear. First, the pulses traveling through the fast light medium (dashed) are significantly advanced relative to the vacuum pulses (solid), as can be seen by the clear separation of their leading edges. Even more interesting, though, is that the two different symbol shapes can be distinguished no sooner for the fast-light pulses. Before approximately $-30 \text{ ns}$ the two symbols cannot be distinguished for either the vacuum pulses or the fast-light pulses. If the information were traveling at the group velocity, we would expect the fast-light symbols to separate approximately $t_{\text{adv}} \approx 27 \text{ ns}$ earlier than the vacuum symbols, which is clearly not the case. Even though the general pulse shape of the fast-light pulses overtakes the vacuum pulses, the information encoded on the fast-light pulses goes no faster than $c$.

While Fig. 6.3 is extremely convincing that information travels much slower than $v_g$ for fast-light pulses, it cannot easily be used to quantify the information velocity. To determine the actual information velocity, we turn to the technique developed in the Ch. 5. The resulting bit error rates for the experimental and modeled media (fast-light and vacuum) are shown in Fig. 6.4. The time axes correspond to those used in Figs. 6.2 and 6.3. By considering Fig. 6.4a, we see that the symbols cannot be identified as early as one might come to expect by looking at Fig. 6.3b. Above, I said that the symbols cannot be distinguished before approximately $-30 \text{ ns}$, but Fig. 6.4a suggests that the symbols cannot be accurately identified until later still. The reason for this apparent discrepancy is that Fig. 6.3 gave us the benefit of eliminating noise by averaging, whereas the BER describes the probability of identifying a single pulse, noise and all.

From the data shown in Fig. 6.4, $T_i \equiv t_{c,\text{med}} - t_{c,\text{vac}}$ can be found for both the
Figure 6.3: The primary result of this thesis (along with Fig. 6.6), averaged experimental pulses for the fast-light medium (dashed) and vacuum (solid). These averaged pulses clearly demonstrate that the symbols can be distinguished at approximately the same time for both the vacuum and fast-light pulses. Panel (a) shows the entire pulses while panel (b) shows a zoom of the region where the pulses begin to diverge. The error bars represent the standard deviation of the single-pulse intensity.
Figure 6.4: BER curves for the experimental and modeled fast-light media, and for vacuum. The BER detection threshold is set to 0.1 as a compromise between smaller detection latency $\Delta t$ and reduced uncertainty.
experimental pulses and for the model. The BER detection threshold is set to 0.1 as a compromise between smaller detection latency and experimental uncertainty. Specifically, choosing a numerically small threshold allows for smaller temporal error bars (because the BER curve is more vertical) but at the cost of larger—and possibly more different—detection latencies. Both of these issues—noise and difference in detection latencies—are important, but a compromise must be chosen. There is no clear way to optimize this threshold choice because the specific detection latencies are not know. For example, if the model predicts the detection latency difference $\Delta t_{\text{med}} - \Delta t_{\text{vac}}$ perfectly, then it is advantageous to choose a numerically small BER threshold and thereby reduce the noise. However, it is likely predicted difference becomes less accurate at later times. Using the threshold choice of 0.1, $T_{i,\text{exp}} = 3.4 \pm 1.5$ ns and $T_{i,\text{mod}} = 1.2 \pm 0.8$ ns. The uncertainty in the experimental values comes from the noise and variation in the BER curve itself, whereas the uncertainty in $T_{i,\text{mod}}$ also accounts for confidence in the similarity between model and experiment.

By inserting these results into Eq. 5.15 with $L/c = 1.3$ ns, we arrive at the most important result in this thesis, that $v_{i,\text{fast}} = 0.4(0.7 - 0.2)c$. This measurement represents the first direct measurement of the velocity of information through a fast-light (or any other) medium. As expected from looking at the averaged pulses in Fig. 6.3, the information velocity is much slower than $v_g \approx -0.05c$, and is near $c$. This information velocity is consistent with relativistic causality and supports Chiao’s proposal that $v_i = c$.

Note the unusual error bars, which arise from the propagation of the uncertainty in the $T_i$ measurements through the division in Eq. 5.15. The standard error propagation rule for division comes from the binomial expansion, and is invalid when the uncertainty is on the order of the value itself. In such cases, one must propagate
the uncertainty limits separately. For example, the lower bound \((0.2c)\) is calculated by considering

\[ T_{i,\text{exp}} - T_{i,\text{mod}} = (3.4 + 1.5)\text{ns} - (1.2 - 0.8)\text{ns} = 4.5\text{ns}. \quad (6.4) \]

### 6.3 The information velocity in slow-light optical pulses

In the previous section, I presented the first measurement of the velocity of information. That measurement, conducted for pulses traveling through a fast-light medium, helps resolve the long-standing controversy of whether information can travel faster than \(c\) on fast-light pulses. It also supports Chiao’s proposal that \(v_i = c\). However, many researchers (if not most) expect that information will not travel faster than \(c\). The prediction that truly makes Chiao’s proposal unusual and contentious is that it also suggests that \(v_i = c\) for slow-light pulses [11], whereas most researchers expect that \(v_i = v_g\) when \(v_g < c\) [16, 50, 54, 55]. In order to resolve this remaining controversy, I bring the \(v_i\)-measurement technique described above to bear on the information velocity of slow-light optical pulses.

Figure 6.5 shows examples of single pulses after propagating through (a) vacuum, (b) the slow-light medium, (c) numerically modeled vacuum, and (d) the numerically modeled slow-light medium. Again, the time origins of the experimental and modeled pulses cannot be directly compared because the origin is arbitrarily chosen for the experimental pulses. For the modeled pulses, time \(t = 0\) corresponds to the time information was encoded. The pulse powers are the same as described in Sec. 4.2.2, with the later half of the pulses either increasing or decreasing in
intensity.

To isolate the effects of propagation, we again observe the averaged pulses. Figure 6.6 shows all four types of experimental pulses, with the 50 pulses of each type averaged together. The qualitative similarity to fast light case (Fig. 6.3) is immediately clear; although the slow-light pulses (dashed) are significantly delayed relative to their vacuum counterparts (solid), both slow-light and vacuum symbols can be distinguished at approximately the same time. This clearly demonstrates that the information does not travel at the slow-light group velocity; if it did, we would be able to distinguish the vacuum symbols \( t_{del} \approx 68 \, \text{ns} \) earlier than the slow-light symbols. This seems to support Chiao’s proposal that information does not travel at the group velocity, but at \( v_i = c \).

Like in the previous section, the results shown in Fig. 6.6 can be quantified using the technique described in Ch. 5. The resulting bit error rates for the experimental and modeled media (slow-light and vacuum) are shown in Fig. 6.7. The time axes correspond to those used in Figs. 6.5 and 6.6.

From the data shown in Fig. 6.7, \( T_i \equiv t_{c,med} - t_{c,vac} \) can be found for both the experimental pulses and for the model. The BER detection threshold is again set to 0.1 as a compromise between smaller detection latency and experimental uncertainty. Using this threshold, \( T_{i,exp} = 8.5 \pm 0.5 \, \text{ns} \) and \( T_{i,mod} = 8.0 \pm 2.0 \, \text{ns} \). The uncertainty in the experimental values comes from the noise and variation in the BER curve itself, whereas the uncertainty in \( T_{i,mod} \) also accounts for confidence in the similarity between model and experiment.

By inserting these slow-light results into Eq. 5.15 with \( L/c = 0.66 \, \text{ns} \), we find the other central result of this research, that \( v_{i,slow} = 0.6c \). In the slow-light case, the error bars are slightly more complicated. The fast bound is \(-0.5c\) (which is
Figure 6.5: Examples of single pulses after propagating through (a) vacuum, (b) the slow-light medium, (c) numerically modeled vacuum, and (d) the numerically modeled slow-light medium.
Figure 6.6: The primary result of this thesis (along with Fig. 6.3), averaged experimental pulses for the slow-light medium (dashed) and vacuum (solid). Like in the fast-light case, the symbols can be distinguished at approximately the same time for both the slow-light and vacuum pulses. Panel (a) shows the entire pulses while panel (b) shows a zoom of the region where the pulses begin to diverge. The error bars represent the standard deviation of the single-pulse intensity.
Figure 6.7: BER curves for the experimental and modeled slow-light media, and for vacuum. The BER detection threshold is set to 0.1 as a compromise between smaller detection latency $\Delta t$ and reduced uncertainty.
faster than positive values) and the slow bound is $0.2c$, comfortably excluding the group velocity $v_g \approx 0.01c$ and including $c$. This result, combined with that of the previous chapter, strongly supports Chiao and coworkers’ proposal the $v_i = c$ for regardless of the group velocity.

### 6.4 Uncertainty and experimental improvements

There are a number of ways that the experiment presented here can be improved. Perhaps the most obvious way is to improve the equipment. Faster and less noisy detection equipment will result in less noise in each pulse, and therefore less uncertainty in the BER. The number of sample waveforms of each type (50) was restricted by the need to limit the overall frequency drift in the laser. Stabilizing the laser further would allow more waveform acquisitions and therefore increased certainty in the results. Also, improvements in the pulse generation and shaping system will allow for sharper “discontinuities” on the pulses, directly reducing the detection latency. In these experiments, the biggest limitation of this sort was the arbitrary waveform generator, which has a rise time of approximately 40 ns. Next was the double-passed acousto-optic modulator (AO), which was slightly slower and more difficult to model than a single-passed AO.

One simple way that the experimental uncertainty can be reduced is by using a longer medium. According to Eq. 5.15, a larger $L$ will reduce the fractional uncertainty in the denominator, and therefore in $v_i$. The only cost of this choice is that the group velocity will be less extreme (that is, it will be closer to $c$) for a given group delay $t_g$.

Finally, improved modeling of the experiment will help better gauge the effects of the detection latency. The biggest aid to the model will be the replacement of
the double-passed AO, which has very complex filter characteristics.

6.5 Discussion

The experimental results presented in this chapter, the capstone of my research and this thesis, strongly suggest that the velocity of information is independent of the group velocity. In both the cases of fast and slow group velocities, the information velocity is measured to be approximately $0.5c$ but includes $c$ within the error bars. This result strongly supports Chiao’s proposal that $v_i = c$ and is consistent with the special theory of relativity.

Perhaps the most unsettling aspect of Chiao’s proposal is the problem of detecting a point on a waveform. In the pure mathematical sense, one needs only measure an infinitely small part of the waveform after the discontinuity to have all of the information about that waveform (up the the next discontinuity, at least). However, this is not true in the presence of noise, and so not experimentally possible. The observation must occur for some finite time and will necessarily include noise and other distortions (from the detection equipment itself, for example). The similarity between this delay and the detection latency may not be superficial. The detection latency is, in my opinion, the concept needed to extend Chiao’s proposal to become a physically quantifiable theory.
Chapter 7

Conclusions

To conclude this thesis, I’d like to step back and consider the significance of the work presented here. This is not a summary of the science itself, which is contained in the introduction, but a summary of the role and implications of the science. Specifically, I hope to address questions of why my work is significant, where it fits in the larger field of information velocity research and information transmission, and finally, what remains to be done.

7.1 The significance of this research

Outlining the significance of several years of research is always a difficult and contentious task, but I believe my work is important for two major reasons. First, it represents the first formulation of an information-theoretic technique for measuring the information velocity. Second, my work includes the use of this new technique to measure $v_i$ in the cases of fast- and slow-light pulse propagation. I find that in both the fast- and slow-light cases, the information propagates at $v_i \sim c$ rather than at the group velocity.

I believe that the development of this framework for measuring the information velocity is of fundamental significance. Specifically, the transmission and identi-
CHAPTER 7. CONCLUSIONS

The classification of an alphabet of distinct symbols is a completely new approach to the information-velocity problem. Using this new symbol-based approach, it is now possible to directly measure the information itself. Previous research has been devoted to measuring arbitrary physical or mathematical features that have tenuous connections to information, at best. My technique focuses on no such feature—or rather, no single feature—but measures the velocity of the information itself. This will allow future experiments to truly address the heart of the matter and focus on information.

Another advantage of this symbol-based approach is that it provides an elegant way to incorporate noise and distortion: the detection latency. I believe the definition of the detection latency and the characterization of its role in information transfer is also a new and important result.

As stated in the introduction to this thesis, I believe this technique can be used not only to answer fundamental questions about the physics of information transmission, but also to help us understand and improve the technology of information transmission. Whether such exploration will prove useful, or even occur, I cannot say. However, if there is interest in optimizing the velocity of information transmission, then this technique can provide a useful diagnostic tool.

I have not just designed this measurement technique, though; a measurement technique is of no use if it is never implemented. I have brought this technique to bear on two important cases; information on pulses traveling faster than $c$ and information on very slow pulses. The question of information on fast-light pulses has been plaguing the community for a century. While few are surprised by the result that information did not travel faster than $c$, arrival at that result via a carefully constructed and information-centric approach brings us much closer to resolving the
issue completely.

The other experimental result—that information travels much faster than $v_g$ on slow-light pulses—may be the more fascinating result. It raises many more questions about optimized communications and computing, and about how we think of digital communication techniques.

### 7.2 The role of this work in the field of information velocity research

There has been a great deal of research devoted to the information velocity, and I do not mean to dismiss its importance or usefulness. My work does not supersede this research. Rather, my work fills a hole in the field and connects previous works. There have been many other important results that relate to—and have influenced—my own research. The seminal work of Sommerfeld in Brillouin effectively created this field of study. It has stood for a century and is still useful today as a thorough theoretical examination of pulse propagation [2]. It did not, however, connect this knowledge of pulse physics to the true propagation of information in a general way.

Recently, Chiao and coworkers have formulated a theory [11] of information velocity that is closely related to Sommerfeld’s work [2]. Their theory connects information with points of non-analyticity on a waveform and makes a specific prediction that $v_i = c$. However, this theory does not extend trivially to the real physical world where noise and distortion affect points of non-analyticity in extremely complex ways. Yet another piece of the puzzle is provided by many researchers who have shown that it is possible to generate pulses that travel with very little distortion at extremely fast and slow group velocities [7, 8, 13, 14, 26].
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The role of my work amidst all of these other contributions is to provide a true physical connection to information that can be measured. Using the fast- and slow-light techniques that these others have developed and I have refined, I have created pulses similar to those studied theoretically by Sommerfeld and Brillouin, and I have measured the velocity of information on those pulses. This work—specifically the detection latency—provides the physical connection between non-analytic points and information that Chiao’s theory was missing.

7.3 Future information velocity research

The final question I set out to address in this conclusion is the question of what remains to be done. I believe there is important work yet to be done in three areas, which I will call applied experimental, pure experimental, and theoretical. The first of these, applied experimental research, involves the design and engineering of information-transmission devices specifically to optimize the velocity of information. This could include the design of new media, pulses, or detection and identification techniques. It also includes the more basic question of whether such optimization is worthwhile; improvements in information velocity may only be available at the cost of information transfer rate or reliability. If that is the case, many applications will find it more useful to optimize transmission rate rather than velocity.

By pure experimental research on the information velocity, I simply mean further exploration using existing techniques and theories, but with different media and encodings. My experiments with intensity-modulated pulses in Raman-pumped vapors were designed largely for experimental convenience, but there are many other systems that should be explored. For example, does one observe similar behavior with different encoding schemes that employ phase or polarization? Do different
media like optical fibers or coaxial cables behave the same? Also, some fast- and slow-light techniques (see Sec. 2.3.3) may exhibit different information-transmission behavior.

Finally, theoretical research means the acquisition of a better theoretical understanding of the information velocity. The link between Chiao’s theory and the detection latency is promising, but has yet to be explored carefully. Also, there is no good quantum-mechanical treatment of information velocity. The paper by Kuzmich et al. [51] provides a tantalizing connection that may also fit nicely with the detection latency concept, but this avenue must also be explored. Ideally, work such as Sommerfeld and Brillouin’s that describes pulse propagation theoretically can be connected, leading to a more precise understanding of information encoding and propagation.
Appendix A

Modulation instability limits to the bichromatic-Raman fast light technique

The most obvious way to achieve a gain doublet (as described in Sec. 4.1) using Raman gain is to employ a bichromatic Raman pump. If one generates a single beam with two frequency components, it will induce the desired gain doublet. However, in the limit of high gain, the two strong pumping beams interact with each other (via the atoms) to create complex nonlinear effects. In this appendix, I will describe these nonlinear effects and the limitations they impose on fast-light systems that are based on bichromatic Raman pumping.

A.1 High-gain atomic Raman scattering for fast light applications

In an attempt to achieve large relative pulse advancement via bichromatic Raman gain, I have use an experimental apparatus (Fig. A.1) that is capable of producing very large Raman gain. I use potassium atoms because the small hyperfine ground-state splitting ($\Delta_g = 462$ MHz) yields large Raman gain and we use a linearly
polarized drive laser beam that simultaneously optically pumps the atoms into one hyperfine level and drives the Raman scattering process [25]. With this setup, described in greater detail below, I can routinely obtain a gain in excess of $e^{15}$, which should give a relative advancement of $A' \approx 0.45$ according to Eq. 2.51.

Unfortunately, I find that the competing nonlinear optical process known as the modulation instability occurs as the Raman gain is increased. The instability is enhanced when the pump field is bichromatic, and is then often referred to as the cross (or induced) modulation instability (CMI) [6]. In the present context where large atomic coherence is created by the electromagnetic field, the CMI leads to extreme frequency broadening of the Raman pump beams even for low pump beam powers.

To observe and characterize the limitations imposed by this competing nonlinear optical effect, a bichromatic field is passed through a potassium vapor, which is contained in a 20-cm-long glass cell heated to 130 °C, corresponding to an atomic number density of $4 \times 10^{12}$ atoms/cm$^{-3}$. The cell has no paraffin coating on the interior walls to prevent depolarization of the ground-state coherence, nor does it contain a buffer gas that would slow diffusion of atoms out of the pump laser beam. The cell reflects 12% of the incident light at each window.

The atoms are pumped by two laser beams tuned to $\omega_{d\pm} = \omega_{11} + 2\pi(1.67 \text{GHz}) \pm$...
Figure A.2: Level diagram showing the Raman anti-Stokes processes occurring for both pump frequencies. The population is pumped into the upper ($F = 2$) ground state by the same bichromatic pumping beam.

Figure A.3: Total generated power in the orthogonal polarization as a function of input power in each frequency. Closed (open) circles represent data for a bichromatic (monochromatic) beam.
APPENDIX A. MODULATION INSTABILITY LIMITS...

\[ \delta/2, \text{ where } \omega_{11} \text{ is the } ^{39}\text{K } 4^2S_{1/2} (F = 1) \text{ to } 4^2P_{1/2} (F = 1) \text{ transition frequency, as shown in Fig. A.2.} \] 
The pump frequencies are chosen to balance favorably the stimulated Raman scattering, optical pumping, and absorption at the operating temperature. The beams are generated using a continuous-wave Ti:Sapphire ring laser (Coherent 899-21) and two acousto-optic modulators (AOSs). They are recombined and pass through a single-mode optical fiber to remove spatial distortion introduced by the AOs and to ensure that they have the same spatial mode. A polarizing beam-splitter (PBS) is placed after the fiber and the beam is collimated at 450 µm (1/e field radius).

The pump beams perform a dual role in the experiment. They optically pump the atoms into the upper \((F = 2)\) ground state and provide the drive field necessary for stimulated Raman scattering (Raman gain) [25]. Maximum gain occurs for probe frequencies of \(\omega_{\pm} = \omega_{d\pm} + \Delta_g\), as shown in Fig. A.2. Selection rules dictate that light with the orthogonal linear polarization will experience gain [60] for a linearly polarized Raman pump.

The resonance half-width \(\gamma\) of the Raman gain features is weakly intensity dependent for the powers used in the experiment and is approximately equal to \(2\pi(2.3 \text{ MHz})\). We set \(\delta/2\pi = 25 \text{ MHz}\), which is somewhat larger than that required by Eq. 2.47 so that this frequency difference can be more easily resolved with with an optical spectrum analyzer (see below). Because the nature of the modulation instability is found to be insensitive to the choice of \(\delta\) when it is in the range of 10’s of MHz, such an adjustment does not affect my conclusions.

A continuous-wave or pulsed probe field (carrier frequency \(\omega_c\)) can also be injected into the cell with polarization orthogonal to the pump polarization to measure the Raman gain or to measure pulse propagation in the vicinity of the gain doublet.
The probe beam originates from the same laser, travels though a double-pass AO and a single-mode fiber, and is combined with the pump beams via the PBS.

The light emerging from the vapor cell passes through a rotatable linear polarizer and can be sent to a confocal optical spectrum analyzer (Coherent 240, 7.5 GHz free spectral range, 15 MHz resolution) or fast detector (New Focus 1537, 6 GHz bandwidth). The voltage produced by the detector can then be observed with either a fast oscilloscope (Tektronix TDS 680B, 1 GHz analog bandwidth, 5 Gs/s sample rate) or an electronic spectrum analyzer (HP 8566B).

**A.2 Characteristics of the light generated by the modulation instability**

When the bichromatic pump beam is applied to the cell with no probe beam present, the pump light is modified dramatically by nonlinear interactions with the atoms. Figure A.3 shows the power that emerges from the cell in the orthogonal linear polarization (relative to the input pump polarization) as a function of the input pump power in each beam. For low pump power, there is little power converted to the orthogonal polarization, as expected for the case when the effects of the instability are negligible. However, as the pump power increases, an increasing fraction of the total input light is converted to the orthogonal polarization due to the instability. I call this fraction the conversion efficiency. This polarization change is due to the CMI and Raman processes, which can create photons with the polarization orthogonal to the pump polarization [60]. The threshold for this effect (arbitrarily defined as the input power which yields 0.1% conversion efficiency) occurs at approximately 11 mW per frequency component (22 mW total pump
power), which corresponds to a probe beam intensity gain of $e^{12.5}$, measured by injecting a weak probe at $\omega_{d+} + \Delta_g$. Increasing the pump power by a factor of two (gain greater than $e^{15}$) increases the conversion efficiency to 12%.

For comparison, the threshold for a monochromatic Raman pump beam occurs at 25 mW, which is considerably higher than even the total input power for the bichromatic threshold (Fig. A.3). This confirms that the bichromatic nature of the light is an essential part of the effect.

Returning to the bichromatic pump case, the light emerging from the vapor cell in the orthogonal polarization is found to have dramatically modified frequency properties. Figure A.4 shows the optical spectrum of the orthogonally polarized

**Figure A.4:** Optical spectrum of the generated light in the orthogonal polarization with 22 mW in each input frequency. The top scale shows the frequency in units of the ground state splitting.
Figure A.5: Level diagrams showing Raman anti-Stokes scattering (I and II), Raman Stokes/anti-Stokes coupling (III), the CMI (IV and V), and a mixed Raman and CMI effect (VI). In each case, the dotted (dashed) arrows correspond to $(\nu_- + \nu_+)$ photons, and the solid arrows correspond to generated photons.

generated light at the maximum input power of 22 mW per frequency component, as described above. It is seen that most of the generated light occurs near $\omega_{d\pm} + \Delta_g$, the spectral region of maximum Raman gain [60]. There are also notable frequency components near $\omega_{d\pm} - \Delta_g$ and $\omega_{d\pm} + 3\Delta_g$. Note that the peaks are extremely broad compared to the input light and have a series of spikes spaced at the pump beam frequency separation $\delta$.

My interpretation of the observed spectral broadening is that it is due to a combination of Raman and CMI effects. The locations of the broad peaks can be explained by the Raman scattering processes shown diagrammatically in Fig. A.5. The large peak at $\omega_{d\pm} + \Delta_g$ is partly due to the Raman anti-Stokes effect, and the peak at $\omega_{d\pm} - \Delta_g$ is partly due to the Raman Stokes/anti-Stokes coupling [61]. Note that the blue-shifted anti-Stokes line is much stronger than the corresponding
Stokes line because the optical pumping process causes the upper \((F = 2)\) ground state to be more highly populated than the lower \((F = 1)\) ground state. This ground state inversion is different from common thermalized systems that experience gain at the Stokes line, rather than the anti-Stokes line.

The smaller peak at \(\omega_{d\pm} + 3\Delta_g\) is a result of repeated Raman scattering. Light generated by Raman scattering can in turn drive further Raman scattering. Every time a Raman scattering event occurs, a photon is effectively shifted in frequency by \(\Delta_g\), but it is also shifted in polarization by 90°, as described above [60]. Because the apparatus includes a polarizer between the cell and detector, only odd-order peaks (those that have been Raman-shifted an odd number of times) appear. When the polarizer is rotated by 90° so that it is aligned with the input pump light (data not shown), only even orders pass through the polarizer.

Raman processes alone do not completely explain these observations because Raman processes normally produce extremely narrow lines [60], whereas the lines shown in Fig. A.4 are very broad. My interpretation is that the width and shape of these broad peaks are a result of the CMI. The four-wave mixing process that causes the CMI (shown in Fig. A.5) is an efficient mechanism for translating existing frequencies by the pump separation \(\delta\). The process involves two photons of one pump frequency, one photon of the other pump frequency, and a newly generated photon. This process can generate photons that are either shifted up by an amount \(\omega_{d+} - \omega_{d+} = \omega_{d+} + \delta\), (IV of Fig. A.5) or down by an amount \(\omega_{d-} - \omega_{d+} + \omega_{d-} = \omega_{d-} - \delta\), (V of Fig. A.5) [6]. Like the Raman processes, the CMI can also occur repeatedly, creating multiple sidebands. For example, a generated photon of frequency \(\omega_{d+} + \delta\) can interact with two \(\omega_{d-}\) photons to produce a photon of frequency \(\omega_{d-} - (\omega_{d+} + \delta) + \omega_{d-} = \omega_{d-} - 2\delta\). There are also processes that incorporate...
elements of both the Raman and CMI processes, which produce shifts of $\pm \delta \pm \Delta_g$ (VI of Fig. A.5). Because these processes occur repeatedly and shift frequencies in both directions, the otherwise narrow Raman lines become dramatically broadened into a comb with frequencies $\omega_{d\pm} + n\Delta_g + m\delta$ for integer $n$ and $m$ [62, 63]. Due to the resonant nature of the interaction, the power in each frequency must diminish with increasing $|n|$ and $|m|$, as observed in Fig. A.4. This combined CMI and Raman effect is similar to one that has been exploited by, for example, Radic et al. [64] in optical fiber amplifiers to achieve broad flat gain over enormous (22 nm) bandwidths.

To achieve greater frequency resolution, the generated light is focused on the fast detector and the detector output is observed with the electronic spectrum analyzer, revealing frequency differences that are present in the optical field. As seen in Fig. A.6a, the spectrum of the generated photocurrent extends to several GHz and contains broad features at multiples of $\Delta_g$ resulting from the Raman effect. As in the optical spectrum analyzer results (Fig. A.4), there are peaks spaced at the pump splitting ($\delta = 25$ MHz) that are due to the CMI. This is more clearly seen in Fig. A.6b, which shows a smaller frequency range and is recorded with a resolution bandwidth of 1 MHz. Here, the peaks have been labeled according to the optical frequencies that they correspond to. Note that the $\Delta n = 1, \Delta m = 0$ peak occurs at less than the ground state splitting, 462 MHz. The discrepancy is attributed to ac-Stark shifts of the levels. The $\Delta n = 1$ peaks are smaller because they are the results of interference between optical components with mutually orthogonal polarizations, one of which is suppressed by the polarizer.

Figure A.6b also shows that the power between the peaks does not fall to the electronic noise floor, as would be expected for a discrete comb of frequencies.
Figure A.6: (a) Low- and (b) high-resolution electronic power spectra of the voltage generated by the fast detector that senses the light generated in the orthogonal polarization.
Instead, there are no quiet frequency windows over a range of more than 2 GHz. The power remains 20 dB above the noise floor and more than 30 dB above the standard quantum limit in the range from DC to $\sim 1$ GHz. Variation of $\delta$ has no significant effect on the spectrum, which suggests that the presence of this inter-peak power is not simply related to the commensurability of $\delta$ and $\Delta_g$. Harris and coworkers have studied a similar system in deuterium in which they set $\delta = \Delta_g$ and exploit the CMI and Raman generation processes to create very short pulses [62, 63]. Their theory predicts a frequency comb, but does not predict the observed continuous spectrum. This inter-peak power may be related to effects observed by Lukin based on the creation of fast-modulating EIT windows for the driving field [65]. A broad continuous spectrum often indicates chaotic temporal evolution, and chaotic behavior seems quite plausible given that much of the generated light results from instabilities in a highly nonlinear system.

To observe the temporal evolution of the generated light, the output of the detector is recorded directly with a fast oscilloscope as shown in Fig. A.7, where it is seen that the intensity evolves in a complex pattern that roughly repeats with a period of 40 ns. While this corresponds to a frequency of 25 MHz, it should be noted that this is not simply beating between the two pumping beams (which are not transmitted through the polarizer) but is a result of interference between frequency components of the generated field. There is also higher-speed structure (Fig. A.7b) that includes pulses as short as 500 ps, which is at the analog-bandwidth-limit of the oscilloscope.

To determine whether the observed fluctuations are a manifestation of deterministic chaos, the false nearest neighbor and global Lyapunov exponent nonlinear statistical analysis methods are employed [66]. Unfortunately, these tests fail to con-
Figure A.7: Temporal evolution of the light generated in the orthogonal polarization at (a) slow and (b) fast time scales.

verge to a meaningful result due to the limited vertical resolution of the oscilloscope (≤ 8 bits).

These observations depend strongly on the presence of a bichromatic pumping beam and are dramatically different when a monochromatic pump beam is used. With only a single frequency, the Raman process still occurs, but the CMI does not. As a result, the optical spectrum consists of narrow peaks (similar to the input beam in width) spaced by the ground state splitting $\Delta_g$. As expected, the electronic spectrum of the fast-detector signal consists of strong peaks at integer multiples of $\Delta_g$, with the power quickly dropping to the noise floor between the peaks.
A.3 ‘Fast light’ limits due to the modulation instability

To demonstrate the significance of these effects on pulse-propagation, a pulse is injected into a potassium vapor in which a bichromatic Raman pump creates a gain doublet, as described above. The peak separation is set to $\delta = 10$ MHz to satisfy Eq. 2.47 and a 246 ns pulse is used so that the bandwidth is equal to $\delta/5.57$.

To avoid as much as possible the deleterious effects of the modulation instability, the gain is set to $g_0L \approx 7.0$, which is far below the threshold value for the modulation instability described in Sec. A.2. At this gain pathlength, it is expected that $\mathcal{A} \approx 21\%$ and that the pulse distortion due to linear dispersion should be negligible.

Figure A.8 shows the experimentally measured Gaussian-envelope pulse after propagating through vacuum (dashed line) and the theoretically predicted output pulse based on the theory outlined in Sec. 2.1.1 (dotted line). The theoretical pulse is calculated numerically and is exact to all orders in $k(\omega)$, but does not account for nonlinear effects such as the CMI. The solid line shows the experimentally observed output pulse, which is distorted dramatically. The pulse envelope is modulated at 10 MHz, which is equal to the frequency separation of the bichromatic pump beam. The fact that the observed output pulse is very different from that expected by the linear dispersion theory indicates that the CMI is responsible for the distortion, even though the pump beam power is below the nominal threshold for the instability. The instability remains a factor because it is now seeded by probe photons rather than quantum fluctuations, effectively lowering the instability threshold.

Similar pulse-propagation behavior is observed for lower Raman gain, becoming less pronounced only when the gain is so low that the relative advancement is as
Figure A.8: Pulse propagation through the fast-light medium experiencing CMI-based pump distortion (solid line). The pulse is both stretched and modulated at the pump separation frequency, 10 MHz. An identical pulse propagating through vacuum (dashed line) and a pulse as predicted by an exact numerical calculation (dotted line) are shown for comparison.

small as that observed by Wang et al. [7].

A.4 Discussion

These experiments demonstrate that the modulation instability places severe restrictions on the fast-light process. These restrictions are of a fundamentally different nature than those due to linear dispersive effects described previously [31,67]. While the CMI does not directly limit the group velocity, it does restrict the practically achievable pulse advancement to just a few percent of the pulse duration.
Bibliography


[58] A web-based digital filter designer was used to generate the algorithms used. In both cases, the sample rate was 2 GS/s. The program and web site were written by T. Fisher. (http://www-users.cs.york.ac.uk/~fisher/mkfilter/trad.html).


[65] M. D. Lukin. private communication.


Biography

Michael D. Stenner was born in Janesville, Wisconsin on May 23, 1975. Growing up in Fennimore, a town of 2000 in rural Wisconsin, he knew he would study physics before he knew what it was called. In 1993, he graduated from Fennimore High School and matriculated at Lawrence University in Appleton, Wisconsin. There, he studied physics, for which he received his B.A. in 1997. Immediately after graduation, he moved to Durham to enroll in the graduate physics program at Duke University. There, under the supervision of Dr. Daniel J. Gauthier, he received his A.M. in 2001 and his Ph.D. in 2004 for his study of the information velocity in fast- and slow-light pulse propagation.

Publications


M.D. Stenner, D.J. Gauthier, and M.A. Neifeld, “Fast (but causal) information transmission in an optical material with a slow group velocity,” (in preparation).


**Presentations**


“The speed of information in fast light pulse propagation,” Joint Workshop on Photonics in the 21st Century, National Chiao Tung University, Kaung Fu Campus, Hsinchu, Taiwan, (December 2002).


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