Holographic Thermalization

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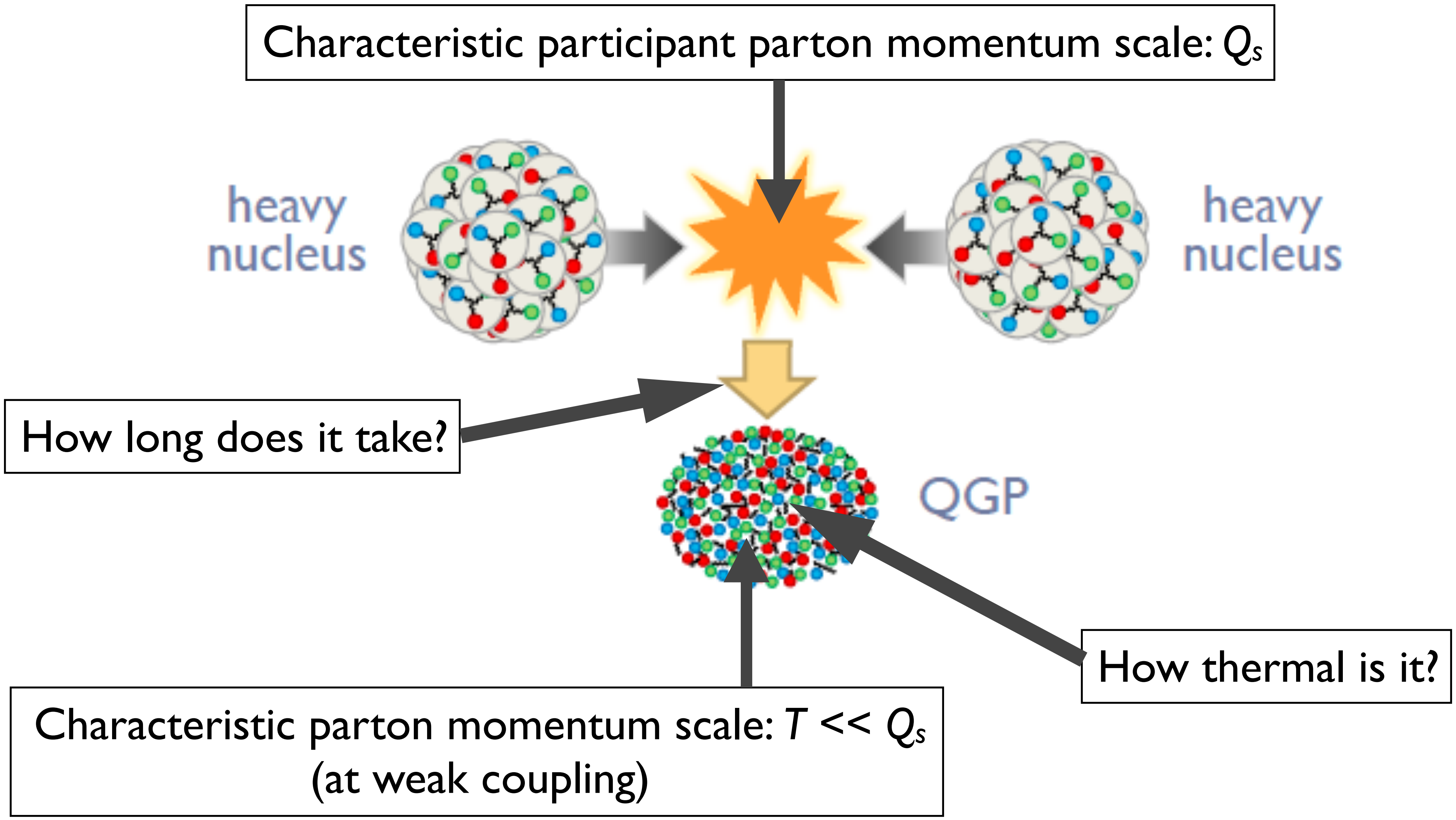
Quark Matter 2011

Annecy, France

Thermalization



How does the thermalization process work at strong coupling?

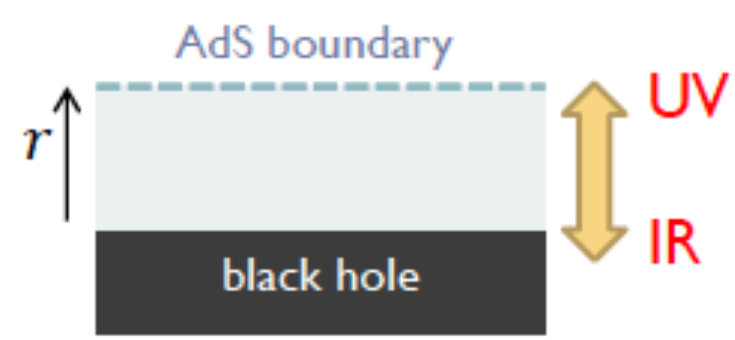
If not "bottom up", what else?

AdS/CFT dictionary

- Want to study strongly coupled phenomena in QCD
- Toy model: $\mathcal{N} = 4\ SU(N_c)$ SYM

$\mathcal{N} = 4$ SYM	holo. dual ⟷ AdS/CFT	String theory in AdS_5
Strong coupling	⟷	Weak coupling
Vacuum	⟷	Empty AdS_5
Thermal state (equilibrated plasma)	⟷	AdS BH
Thermalization	⟷	BH formation
HI collision	⟷	Energy injection

$$\left(\frac{R_{AdS}}{l_s}\right)^4 = g_{YM}^2 N_c \equiv \lambda$$



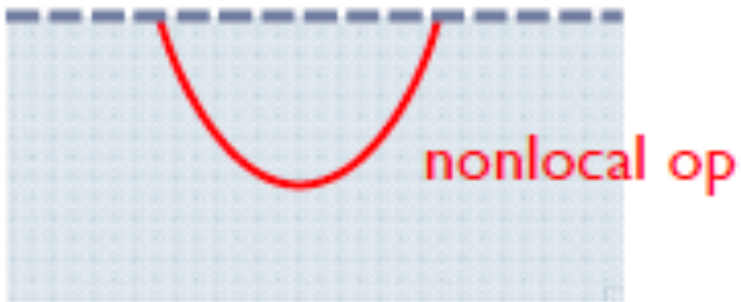
Questions to answer

- What is the measure of thermalization on the boundary?
  - Local operators are not sufficient

$$\langle T_{\mu\nu} \rangle \text{ etc.}$$

  - Nonlocal operators are more sensitive

$$\langle O(x)O(x') \rangle \text{ etc.}$$



- What is the thermalization time?
  - When observables reach their thermal values

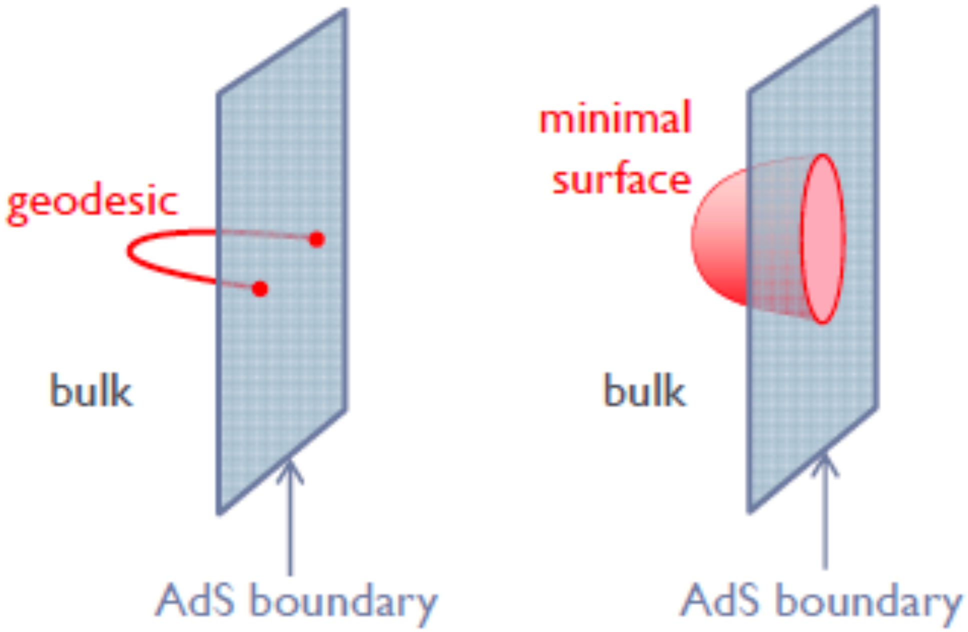
Duke Theory

Thermality probes

- Local operators like $\langle T_{\mu\nu} \rangle$ measure moments of the momentum distribution of field excitations
  - e.g. $\langle k_x^2 \rangle$ vs. $\langle k_z^2 \rangle$
- Nonlocal operators, like the equal-time Green's function, are sensitive to the momentum distribution and to the spectral density of excitations:
  - $$G(\vec{x}) = \int d\vec{k}\, dk^0 \sigma\left(k^0, \vec{k}\right)\left[n\left(\vec{k}\right)+1\right]\exp\left(i\vec{k}\cdot\vec{x}\right)$$
  -
- Entropy is the "gold standard" of thermalization:
  - $S = -\mathrm{Tr}[\rho \ln(\rho)]$ probes all degrees of freedom.
  - Coarse graining mechanism: ***Entanglement entropy***.

Probes we consider

- 2-point function
  - $\langle \mathcal{O}(x)\mathcal{O}(x)\rangle$
  - Bulk: geodesic (1D)
- Wilson line
  - $W = P\{\exp[\int_C A_\mu(x)dx^\mu]\}$
  - Bulk: minimal surface (2D)
- Entanglement entropy
  - $S_A = -\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B[\rho_{tot}]$
  - Bulk: codim-2 hypersurface (same dimension as boundary space)



Use semiclassical approximation

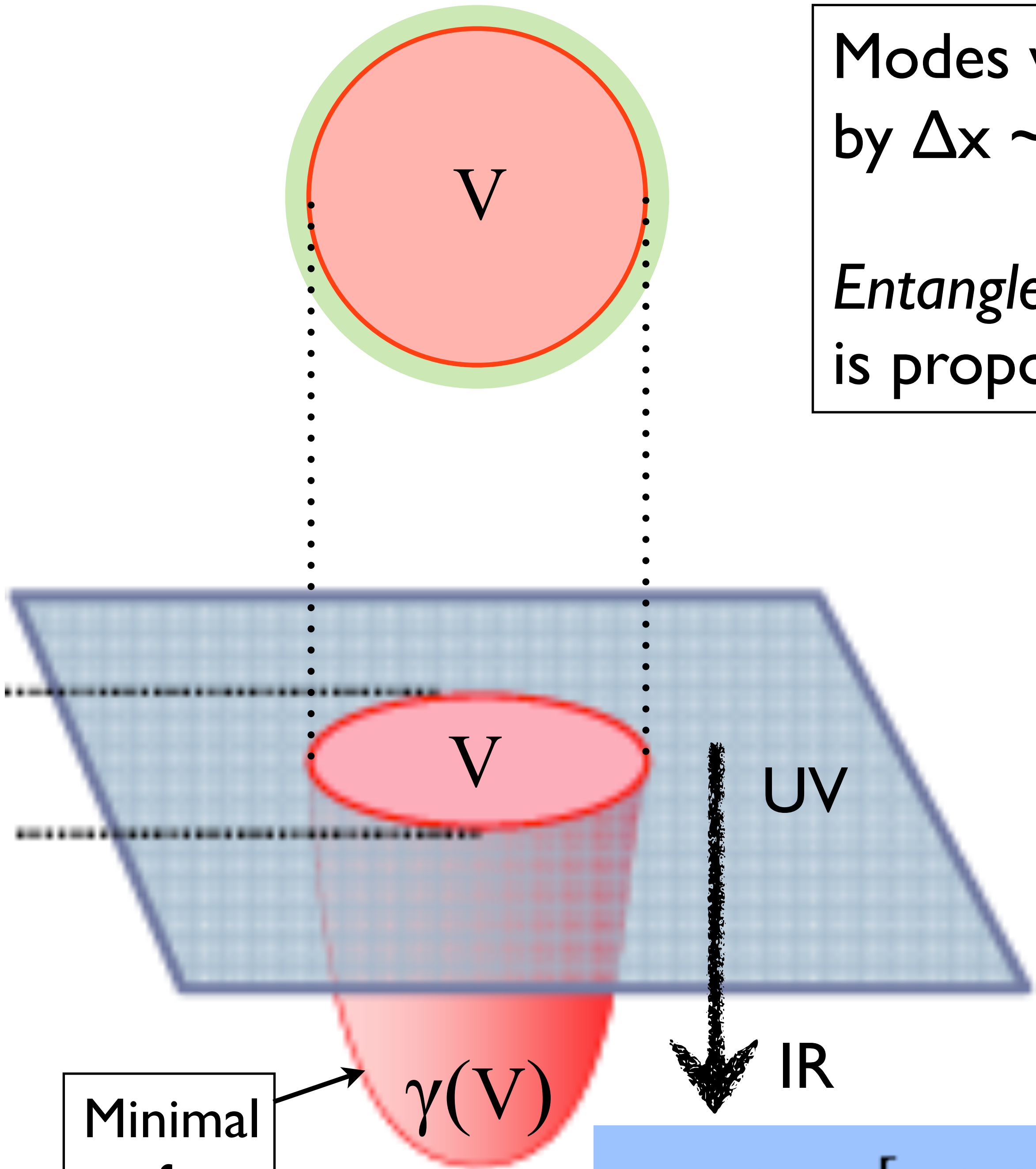
For details: *V. Balasubramanian, et al.*, PRL **106**, 191601 (2011); arXiv:1103.2683

See also: S. Caron-Huot, P.M. Chesler & D. Teaney, arXiv:1102.1073

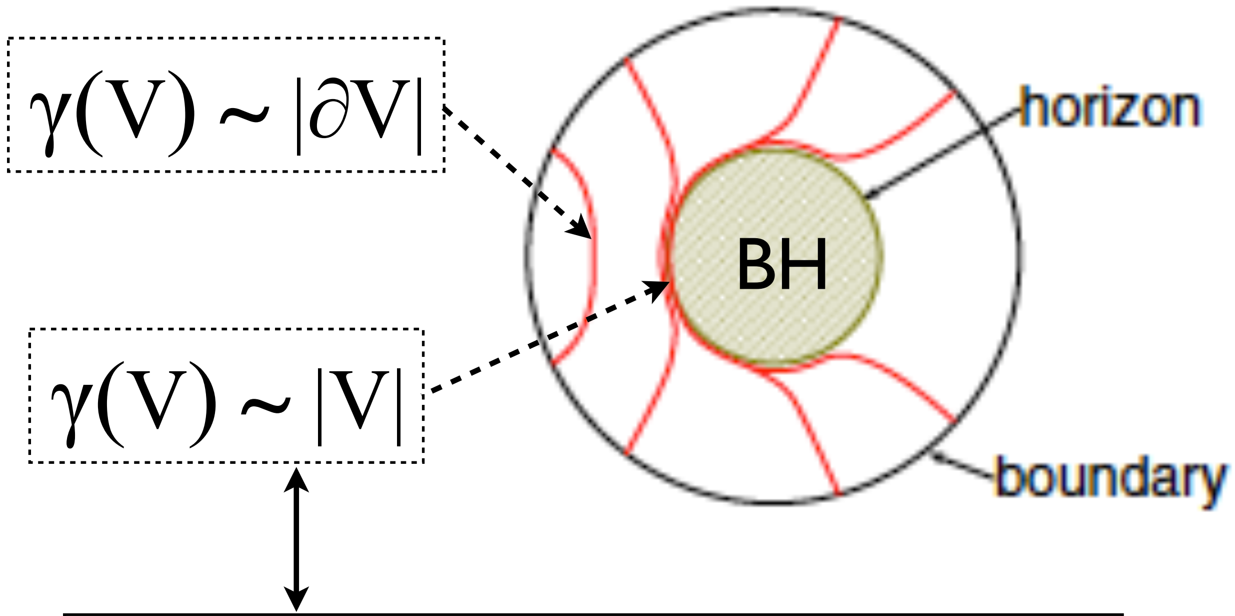
Entanglement entropy

Modes with momentum *k* "leak" into surrounding by Δx ~ 1/*k* ➠ *entanglement* with environment

Entanglement entropy of localized vacuum domain is proportional to surface area (*Srednicki* 1994).



$$S = \frac{\mathrm{Area}[\gamma(V)]}{4G_N}$$



T ≠ 0: *S* proportional to volume
⇔ area of horizon of dual BH

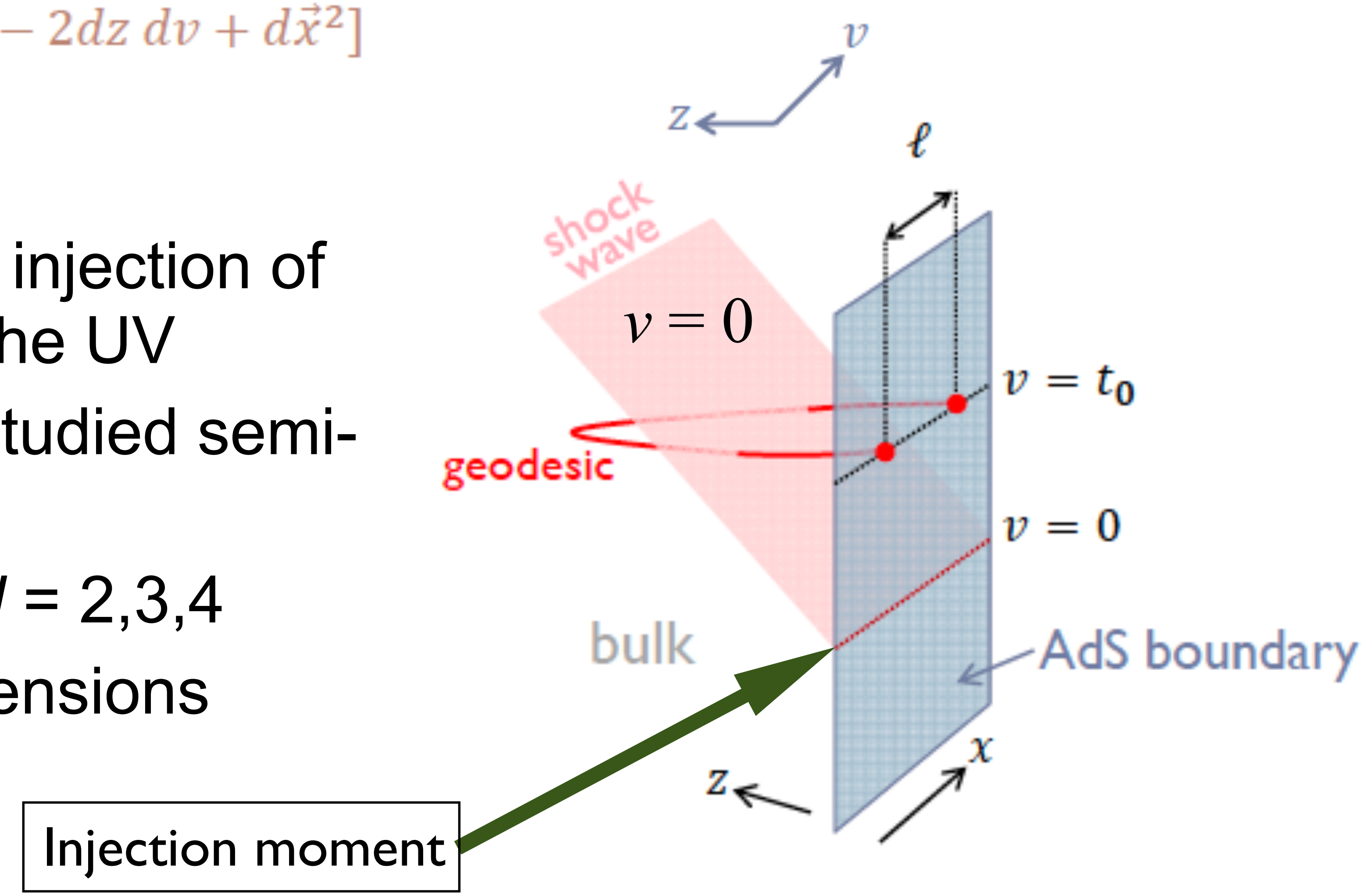
(*Ryu & Takayanagi* 2006)

Vaidya-AdS geometry

- Light-like (null) infalling energy shell in AdS (shock wave in bulk)
  - *Vaidya-AdS space-time* (analytical)

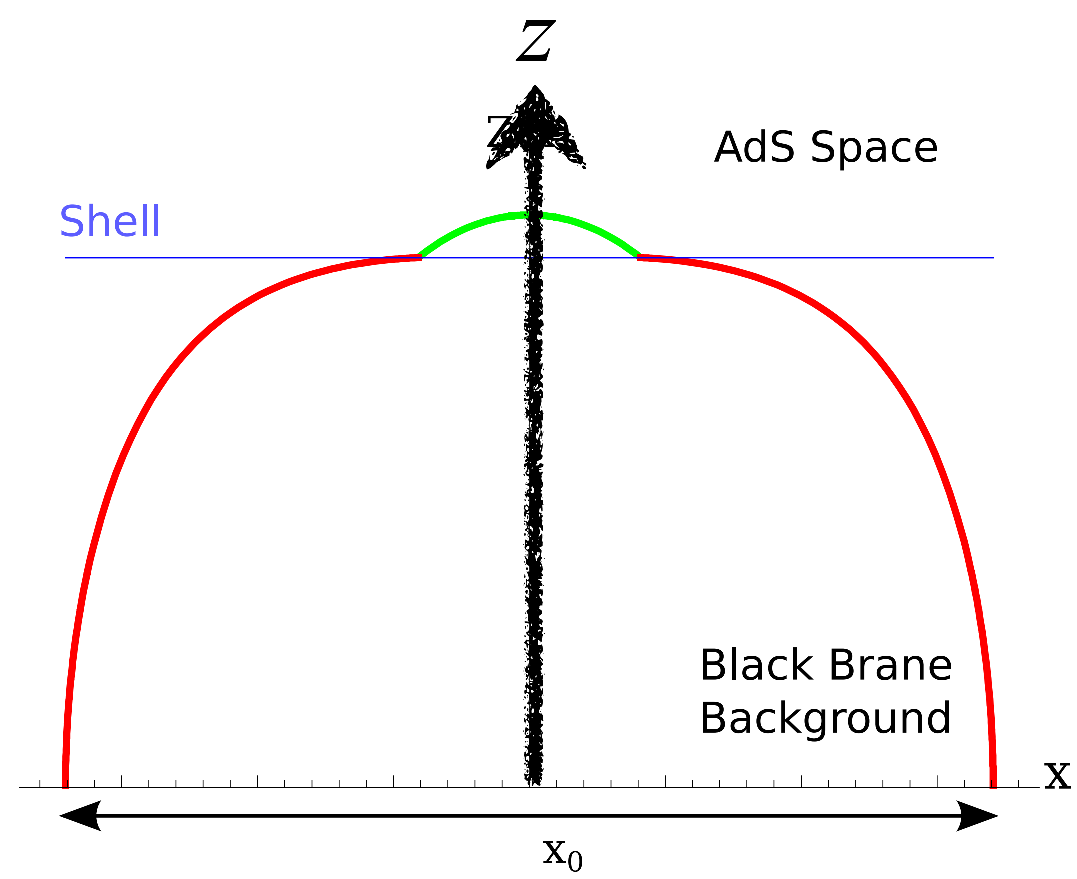
$$ds^2 = \frac{1}{z^2}\left[-\left(1 - m(v)z^d\right)dv^2 - 2dz\,dv + d\vec{x}^2\right]$$

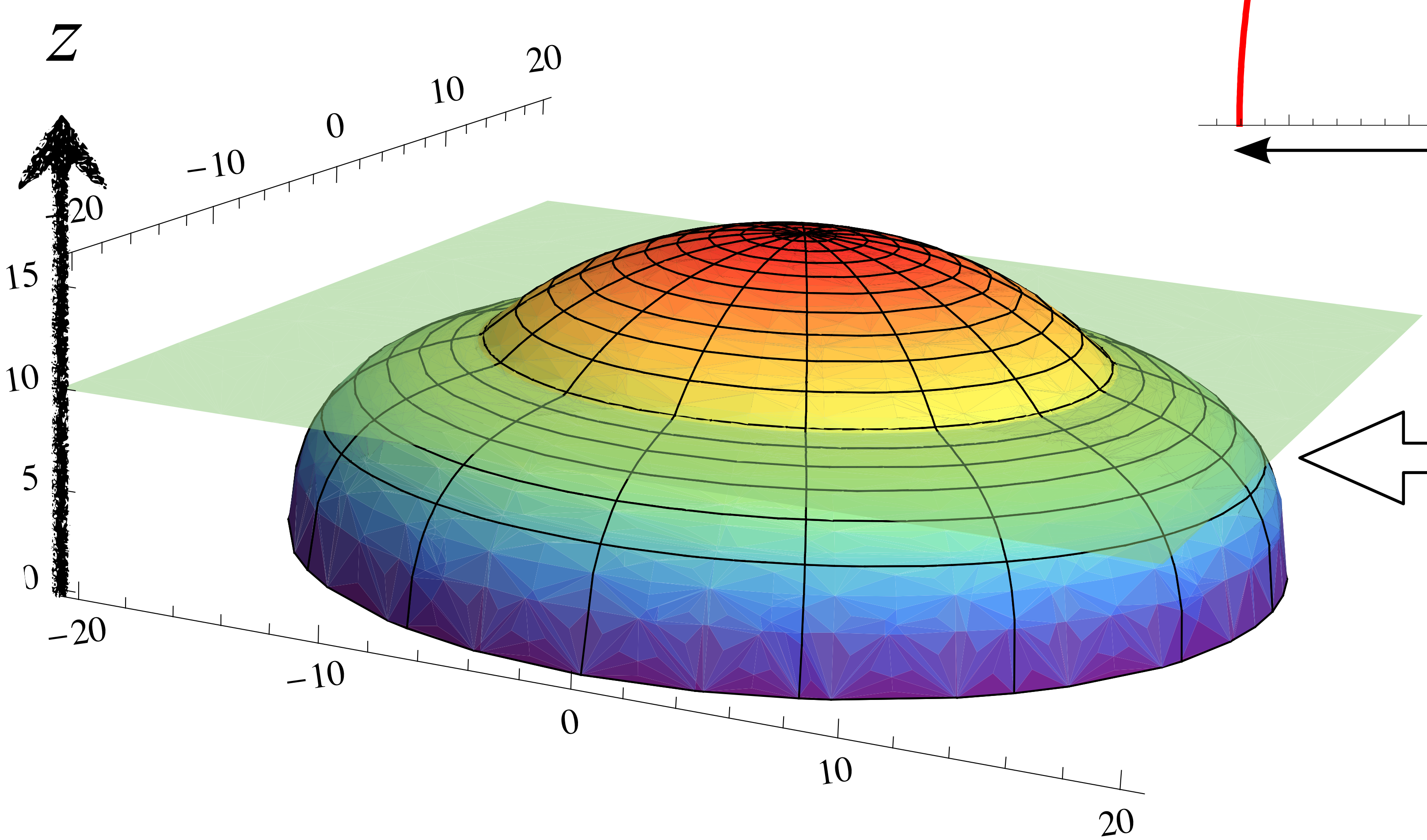
  - $z = 0$: UV $\quad z = \infty$: IR
  - Homogeneous, sudden injection of entropy-free energy in the UV
  - Thin-shell limit can be studied semi-analytically
  - We studied AdS_{d+1} for d = 2,3,4
  - ⇔ Field theory in d dimensions



Examples

Geodesic line "punching" through the falling shell



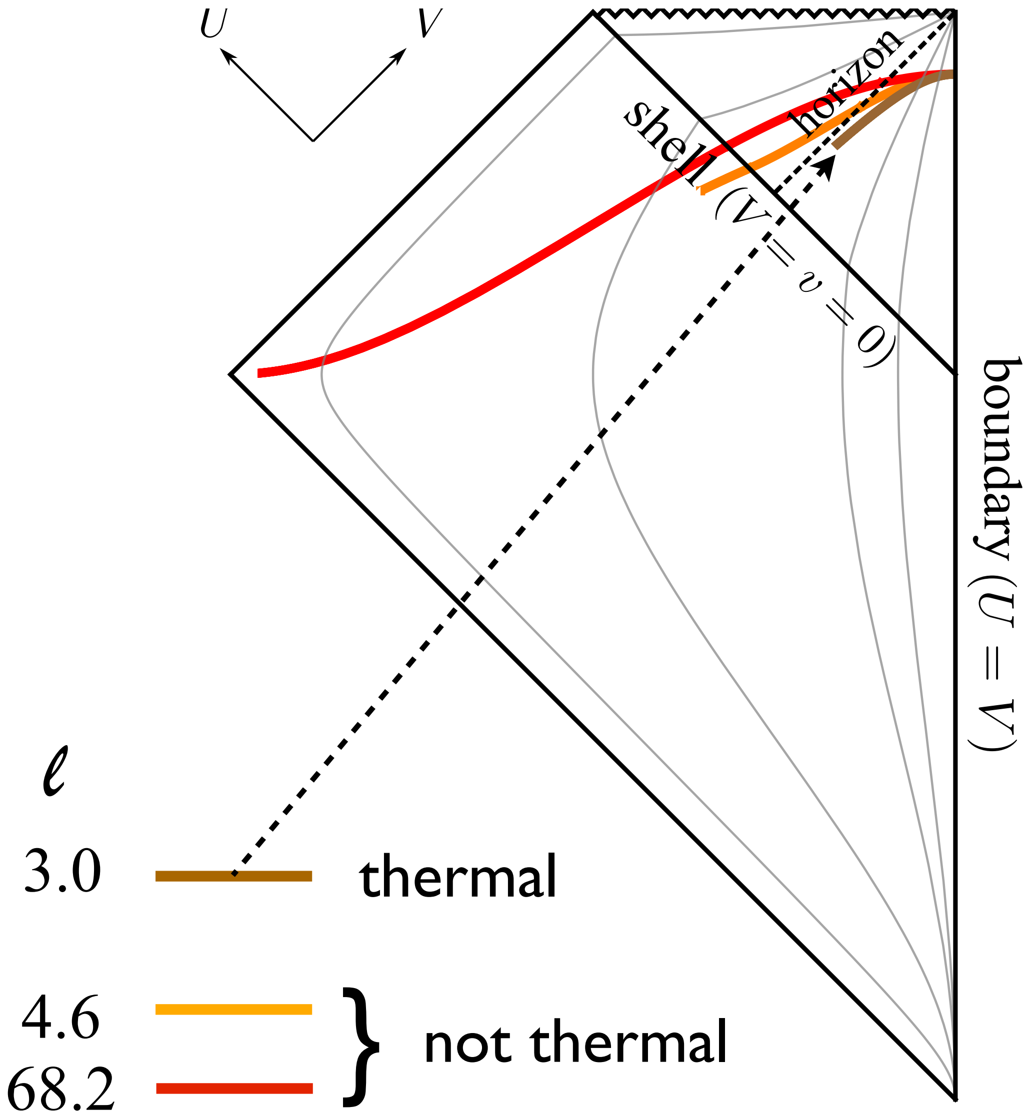


Wilson surface "punching" through the falling shell

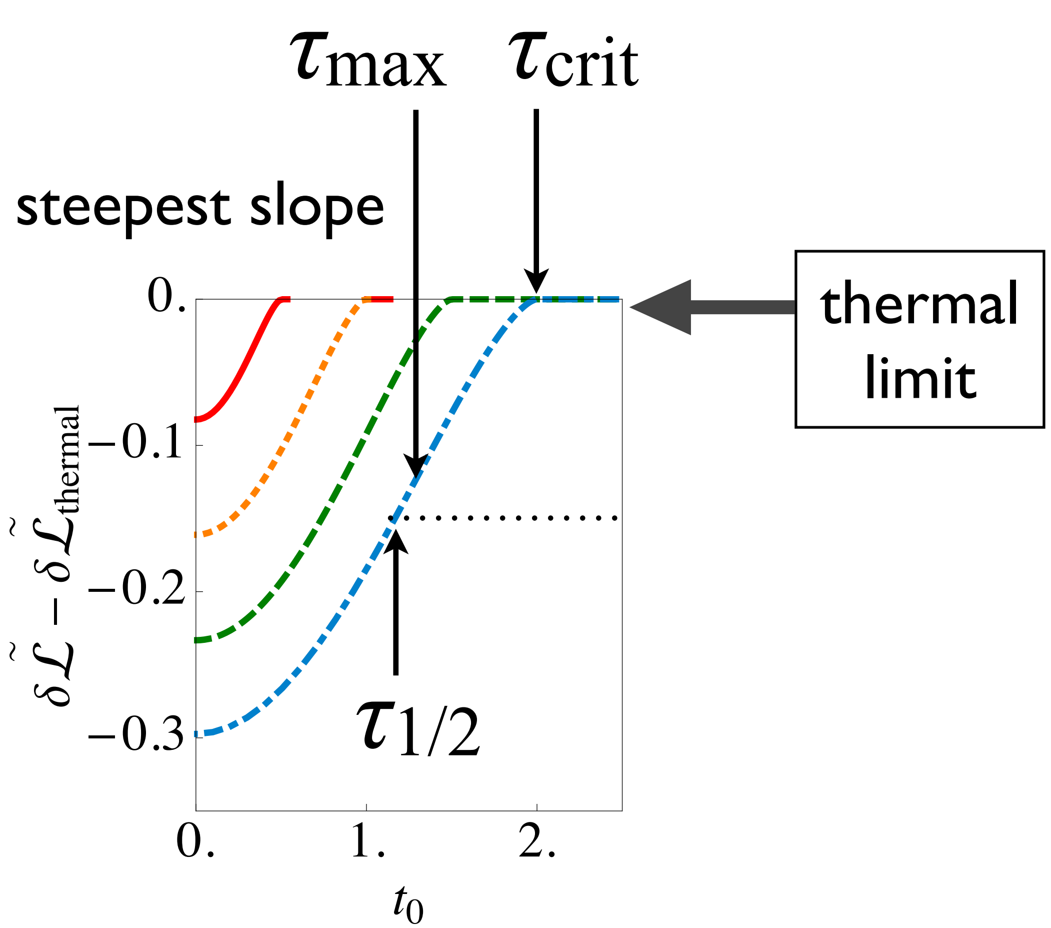
Probing thermalization

Equal-time geodesics for fixed $t_0 = 2$ and $\ell = 3.0,\ 4.6,\ 68.2$

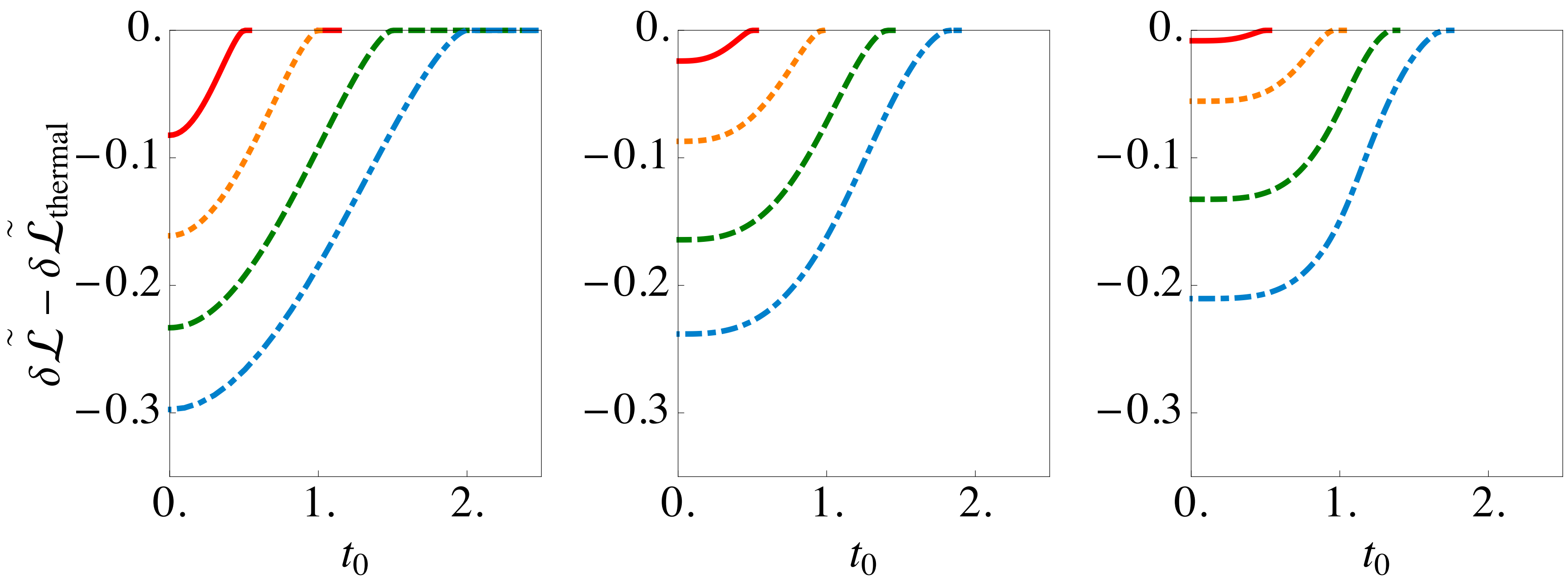
Geodesics staying outside the falling shell only probe "thermal" part of bulk space
➠ 2-point function is thermalized



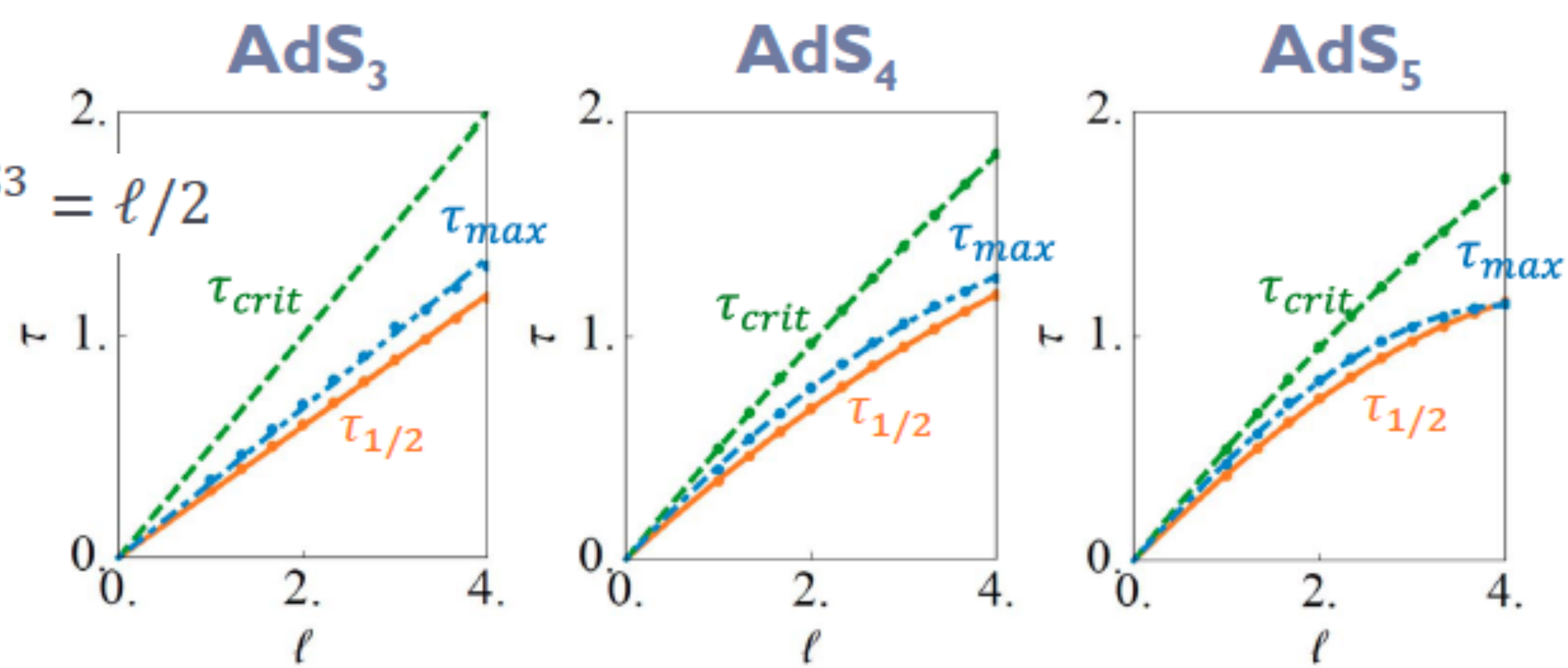
$$\langle O(x)O(x')\rangle \sim \exp[-\delta\mathcal{L}] \quad \text{with} \quad \delta\mathcal{L} = \mathcal{L} - \mathcal{L}_{\text{AdS}}$$



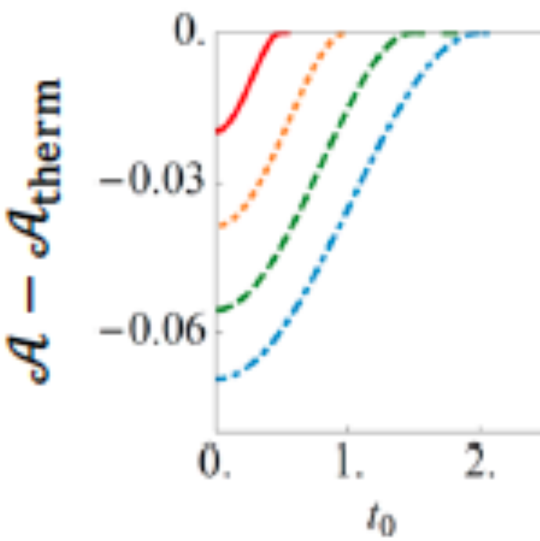
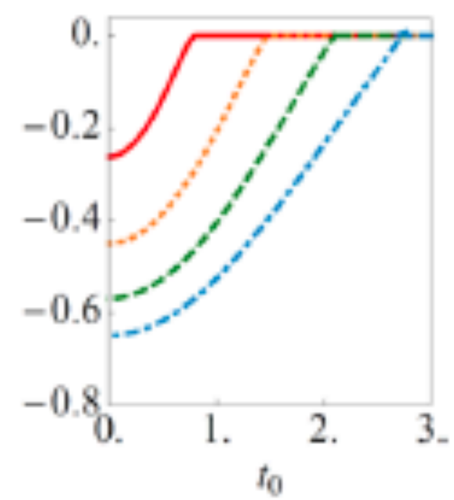
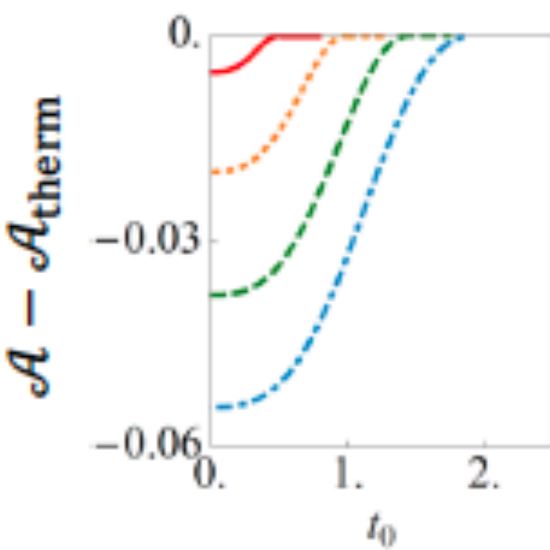
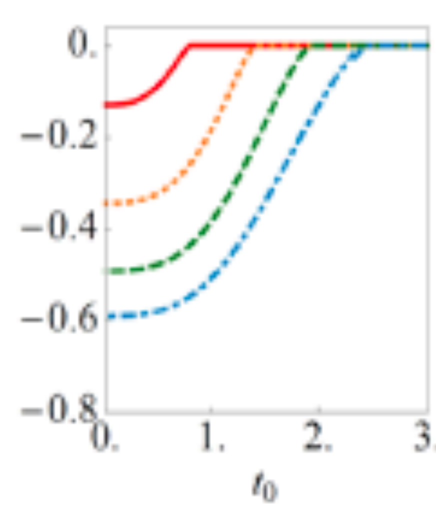
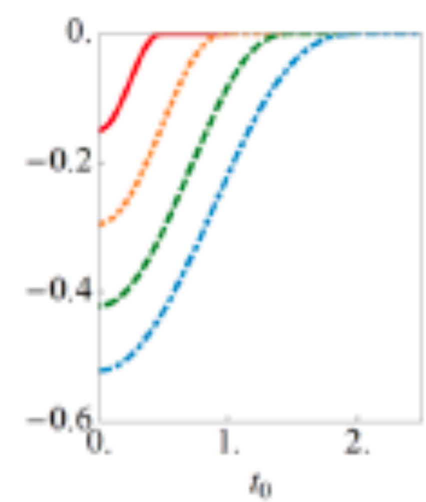
2-point functions



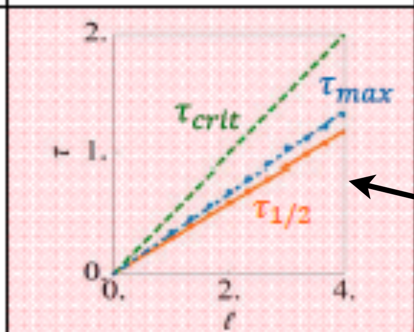
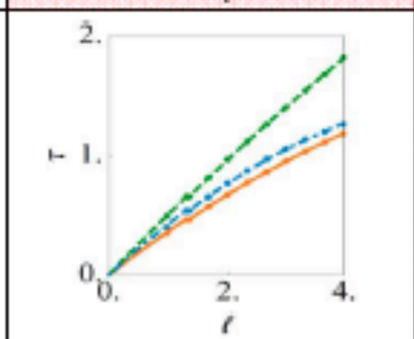
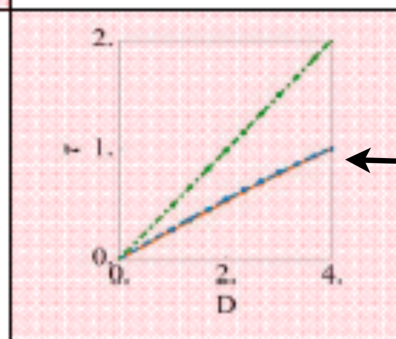
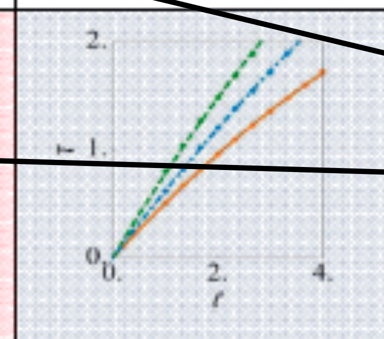
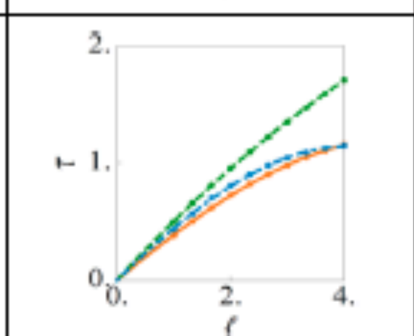
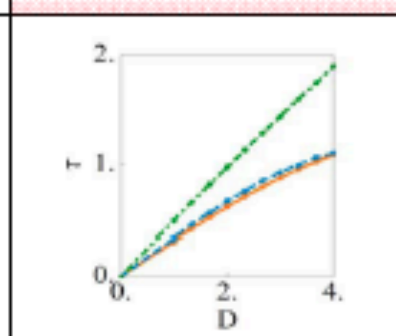
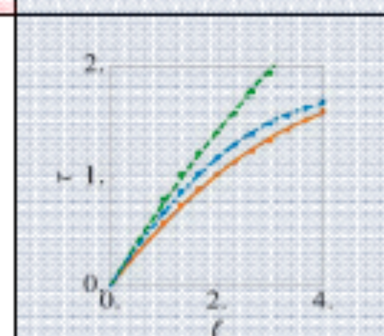
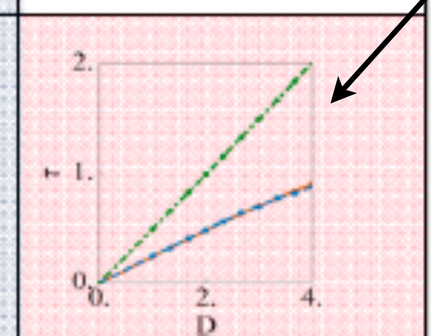
$$\tau_{\text{crit}}^{\text{AdS3}} = \ell/2$$



Higher dim. observables

	Circular Wilson loop	Rectangular Wilson loop	Wilson "sphere"
AdS_4			N/A
AdS_5			

Entropy thermalizes slowest

	geodesic	circular Wilson loop	Rectangular Wilson loop	EE for a sphere
AdS_3		N/A	N/A	N/A
AdS_4				N/A
AdS_5				

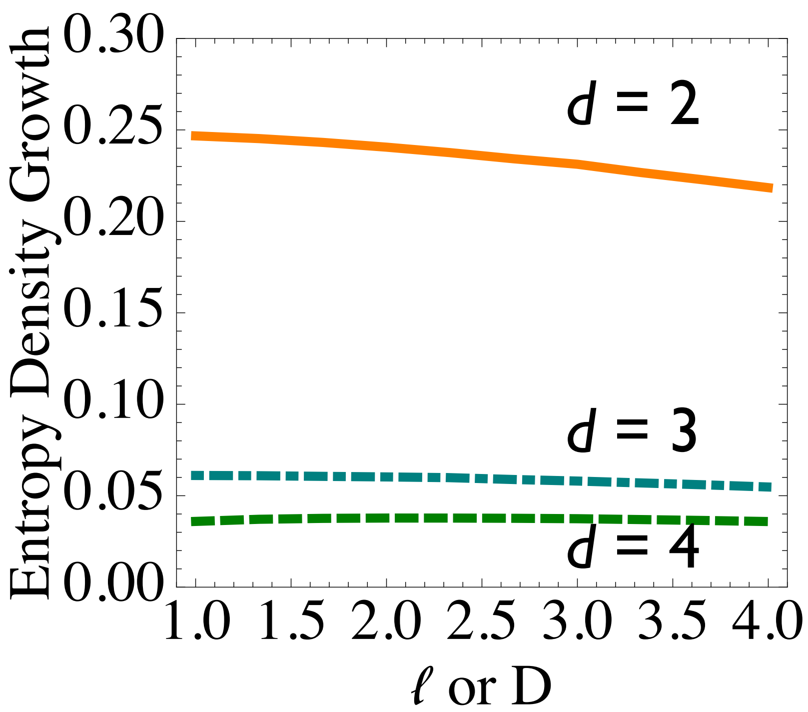
Entanglement entropy of spherical volume in d = 2, 3, 4

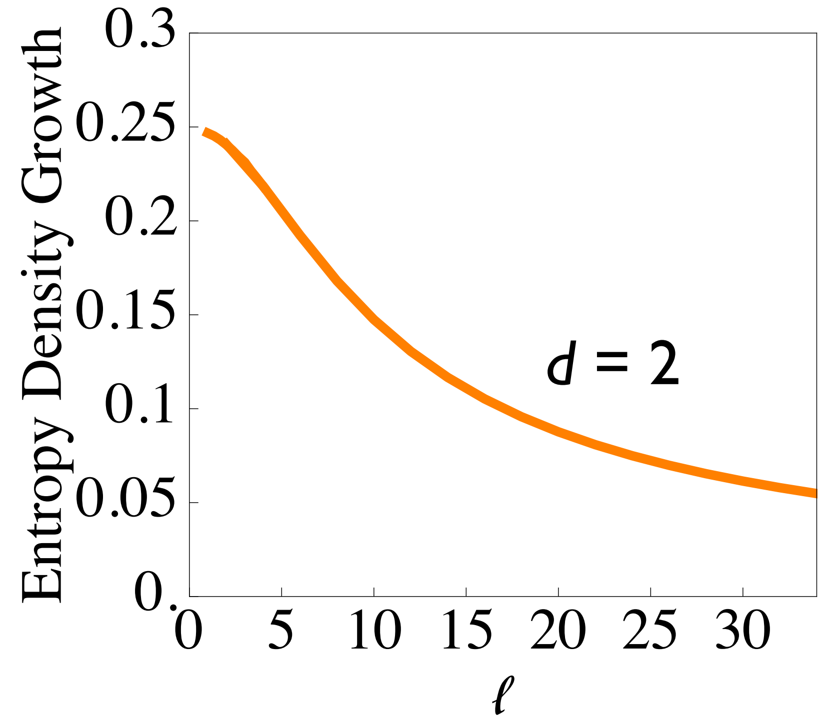
$$\tau_{\rm crit} = \ell/2$$

Thermalization time for entanglement entropy
= time for light to escape from the center of the volume to the surface

Other observables thermalize faster.

Entropy growth rate





Entropy density growth rate is nearly volume independent for small volumes, but slowly decreases for large volumes (numerically difficult to study in $d > 2$).

(Very crude) phenomenology:

$\tau_{crit} \sim 0.5\ \hbar/T \approx 0.3$ **fm/c for** $T = 300 - 400$ **MeV**

Conclusions

- Long-distance observables sensitive to IR modes take longer to thermalize
  - Top-down rather than bottom-up thermalization
- Entropy is the last observable to reach thermal value
- Thermalization proceeds as fast as constrained by causality i.e. at the speed of light
  - True for homogeneous energy injection
  - Speed of sound is expected to govern equilibration of spatial inhomogeneities
- Future research opportunities: Many.
  - See next page....

Outlook

- Compute other observables in the Vaidya model
  - Unequal time correlators; light-cone Wilson loops
- Extend methods to different geometries
  - Colliding shock waves; boost invariant geometries
  - Expanding longitudinal flux tubes
- Extraction of QFT state as function of time
- Entanglement entropy of non-spherical domains
- Beyond the semi-classical approximation
- Non-AdS backgrounds
  - Confining geometries, improved holographic QCD models
- **Whatever else you can think of!**

Je vous remercie

de votre attention