The Re-Combinatorics of Thermal Quarks

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LBNL School on

Twenty Years of Collective Expansion

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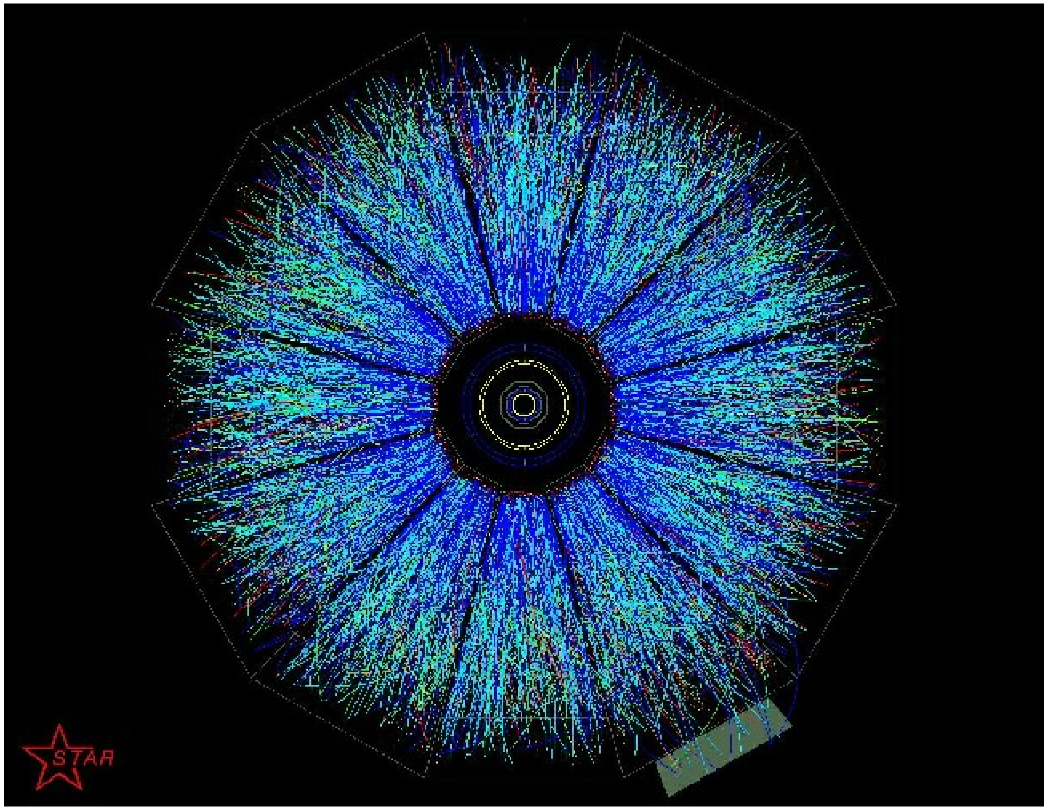
Re-Combinatorics of Thermal Quarks

Special thanks to:

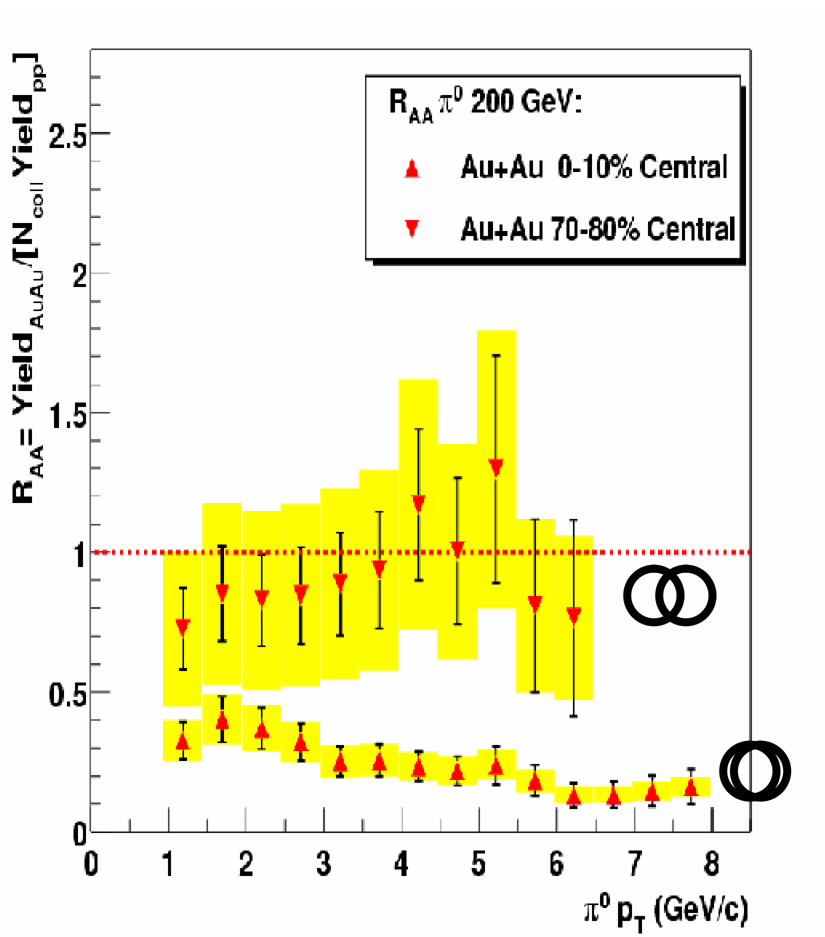
- ***M. Asakawa***
- ***S.A. Bass***
- ***R.J. Fries***
- ***C. Nonaka***

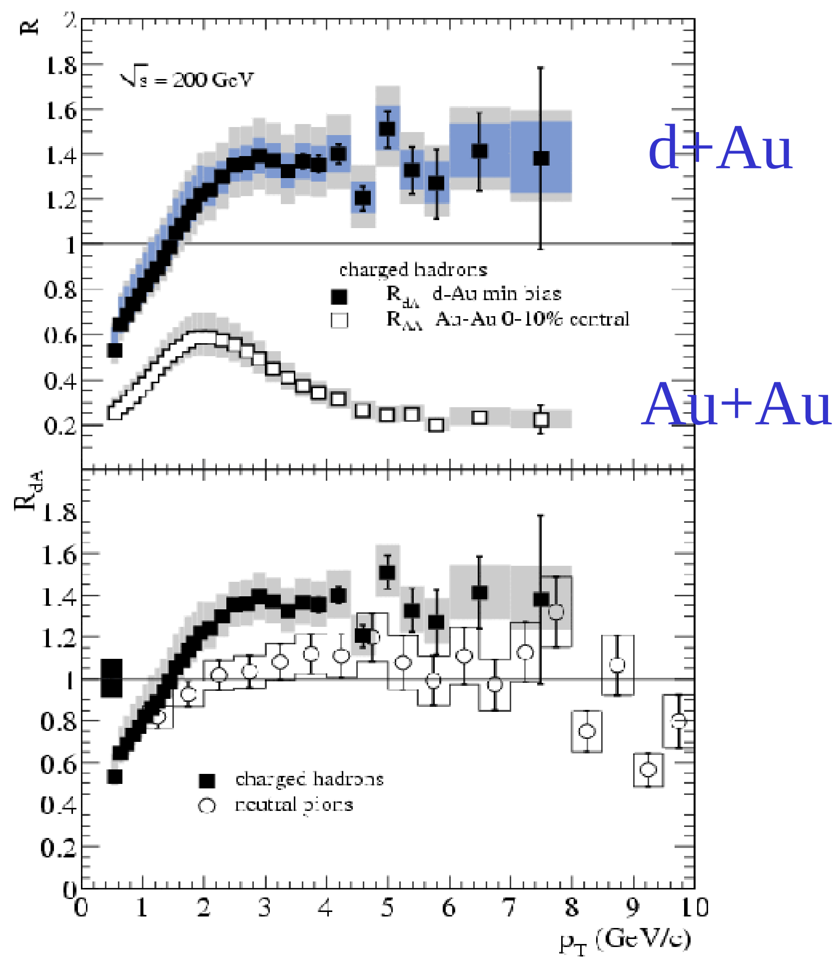
- PRL 90, 202303
- PRC 68, 044902
- PLB 583, 73
- PRC 69, 031902
- PRL 94, 122301
- PLB (in print)



Jet quenching seen in Au+Au, not in d+Au

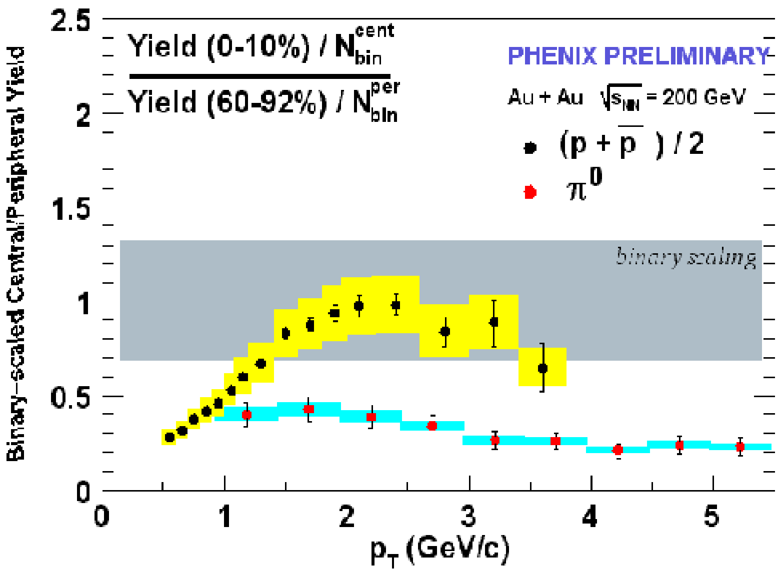
PHENIX Data: Identified π^0

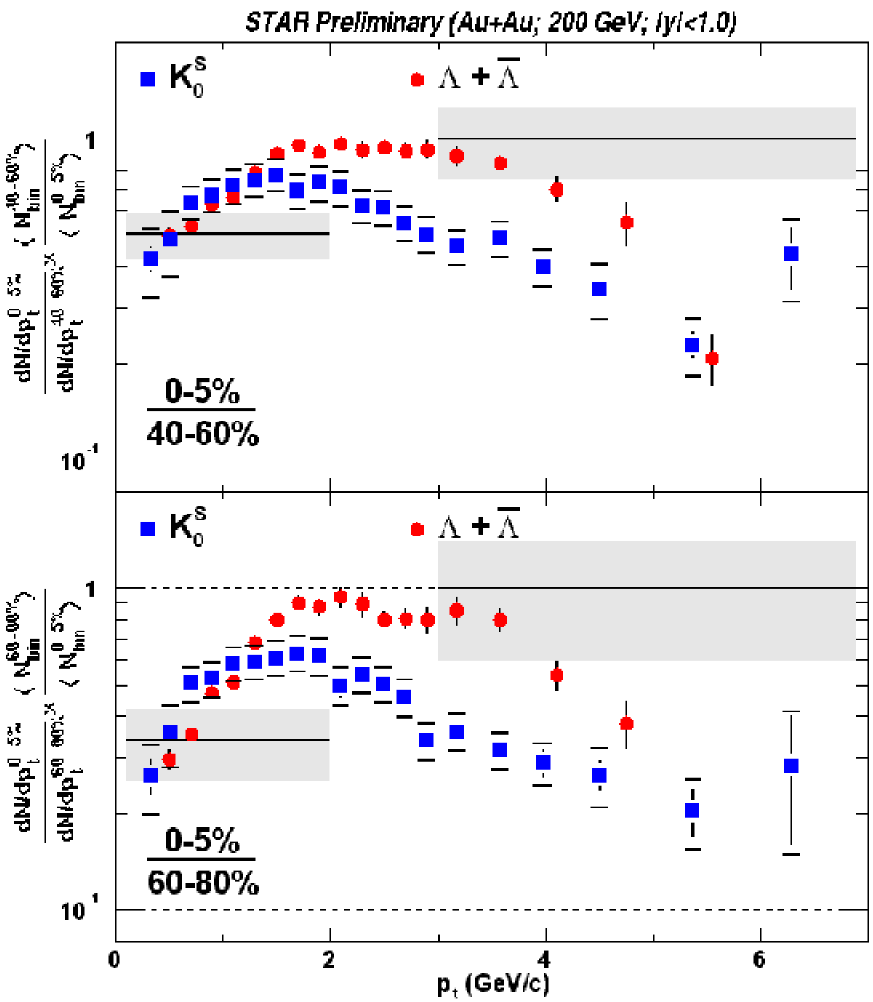




Suppression Pattern: Baryons vs. Mesons

…or what really came as a complete surprise…



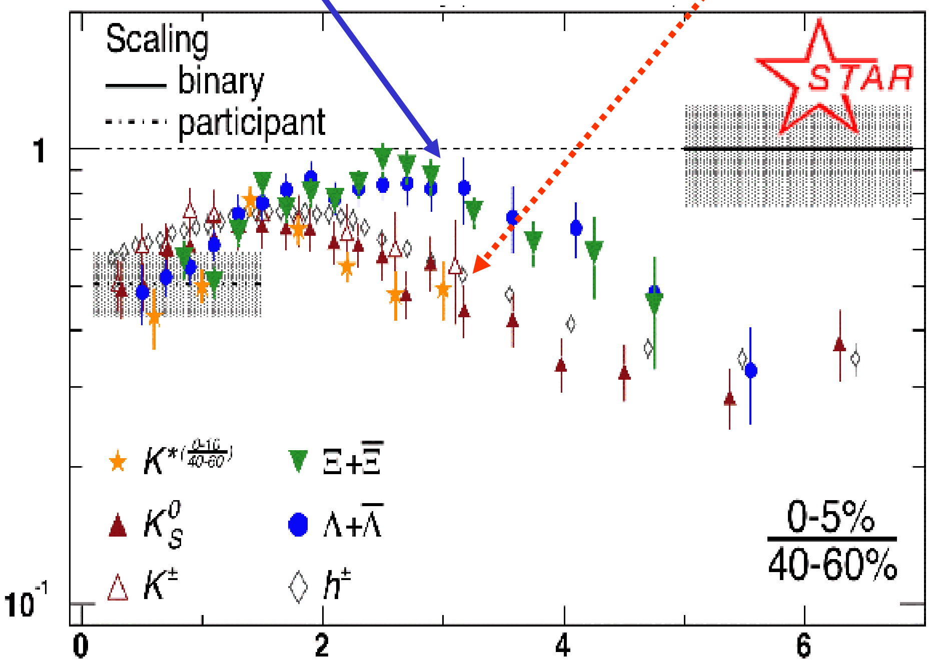


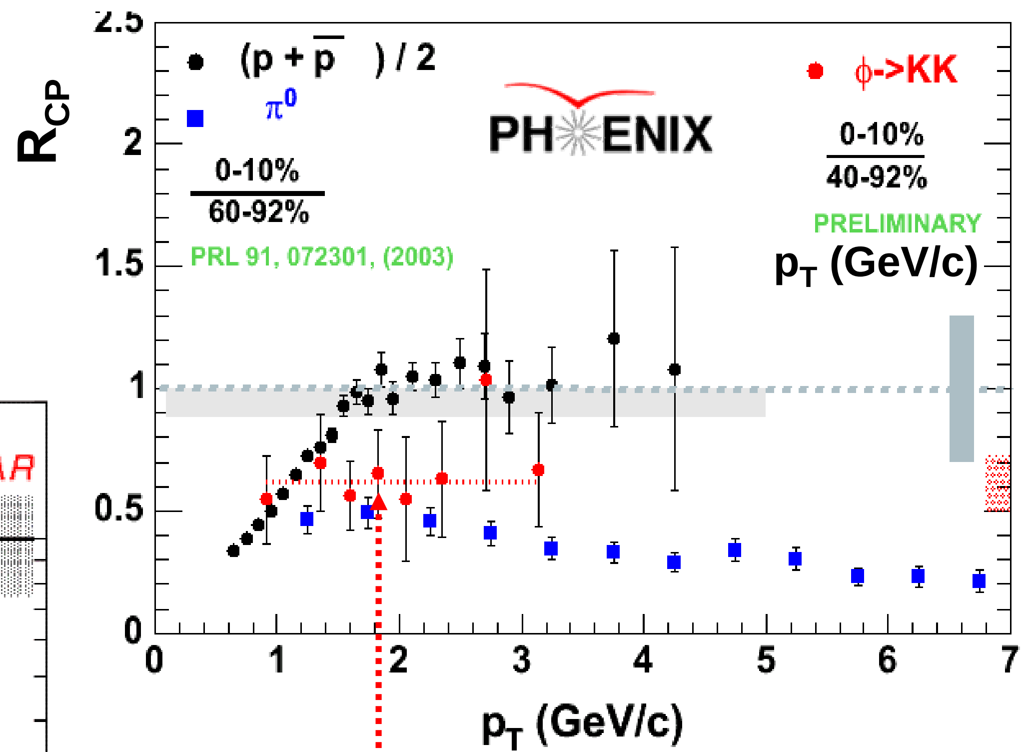
- What makes baryons different from mesons ?

Suppression: Baryons vs. mesons

baryons

mesons



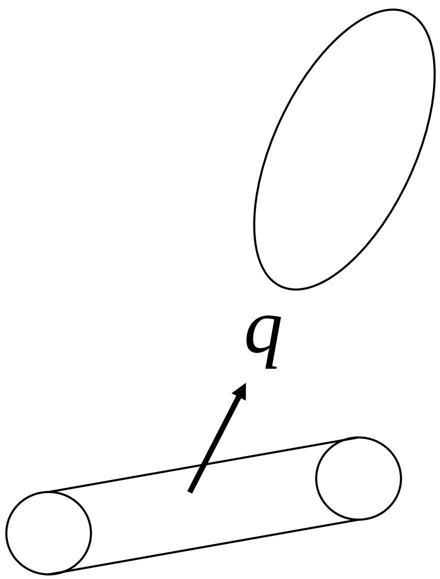


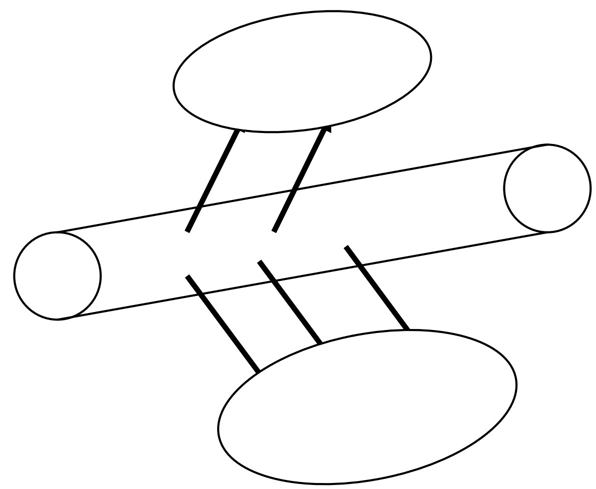
ϕ behaves like meson ?

(also η-meson)

Hadronization Mechanisms

Recombination was predicted in the 1980's – Hwa, Ochiai, …





Recombination

Fragmentation

$$\frac{\text{Baryon}}{\text{Meson}} \ll 1$$

$$\frac{\text{Baryon}}{\text{Meson}} \approx 1$$

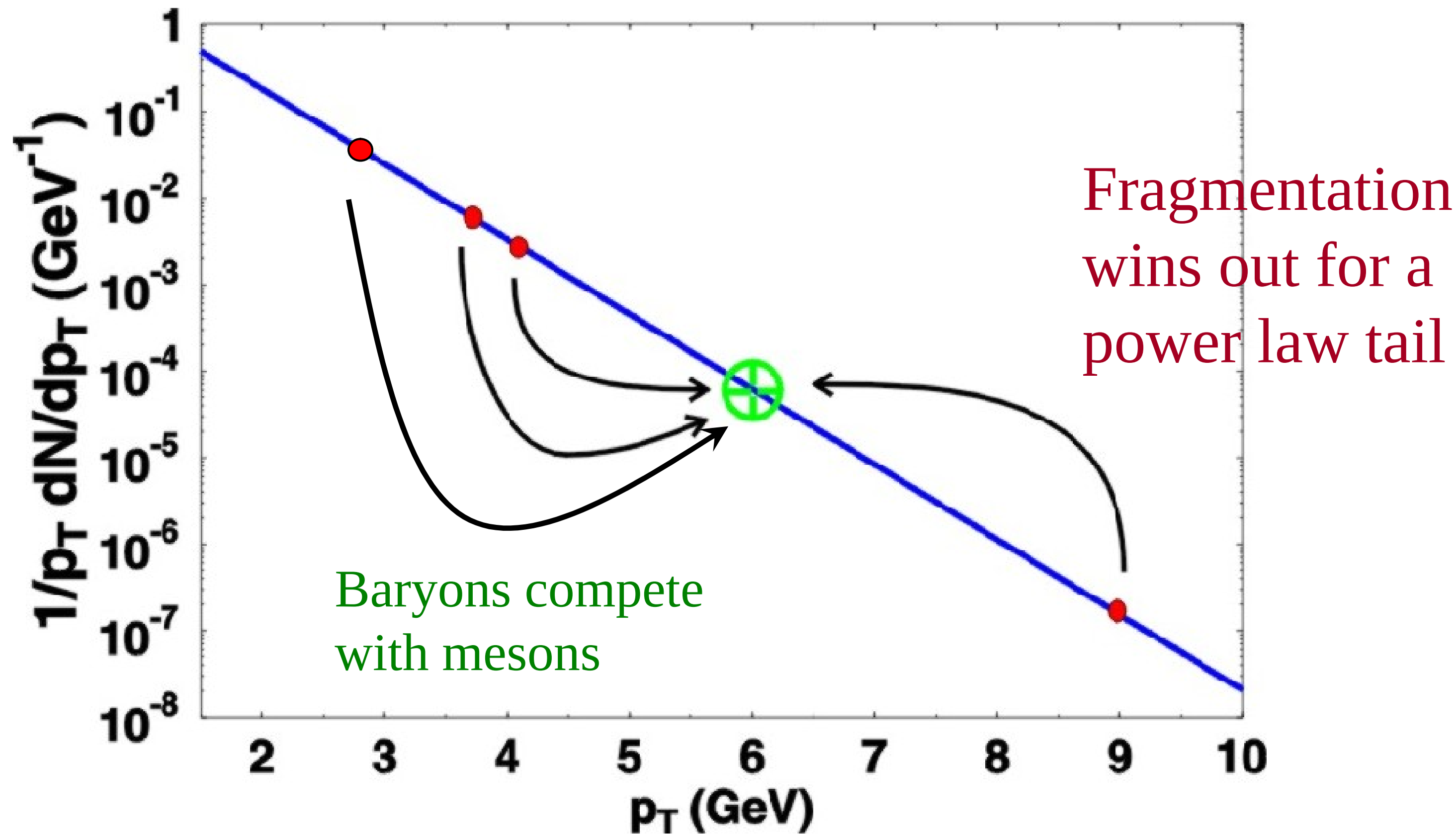
$p_{\mathrm{M}} \approx 2\,p_{\mathrm{Q}} \qquad p_{\mathrm{B}} \approx 3\,p_{\mathrm{Q}}$

S. Voloshin

QM2002

Recombination is favored …

… for a thermal source

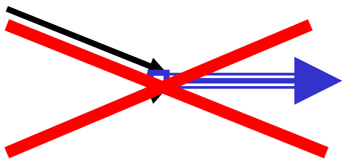


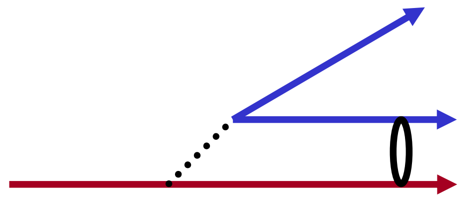
Instead of a History ….

- Recombination as explanation for the “leading particle effect”:
  - K.P. Das & R.C. Hwa: Phys. Lett. B68, 459 (1977)
  - Braaten, Jia, Mehen: Phys. Rev. Lett. 89, 122002 (2002)
- Fragmentation as recombination of fragmented partons:
  - R.C. Hwa, C.B. Yang, Phys. Rev. 024904 + 024905 (2004)
- Relativistic coalescence model
  - C.B. Dover, U.W. Heinz, E. Schnedermann, J. Zimanyi, Phys. Rev. C44, 1636 (1991)
- Statistical recombination
  - ALCOR model (see T.S. Biro’s lecture)
- Quark recombination / coalescence
  - Greco, Ko, Levai, Chen, Rapp / Lin, Molnar / Duke group
  - A. Majumder, E. Wang & X.N. Wang (in progress)

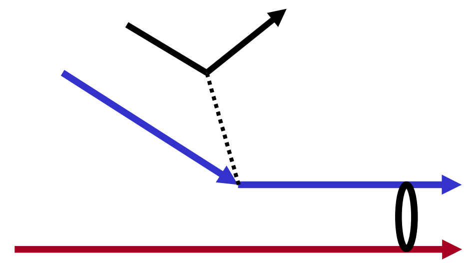
Recombination: The Concept

$p_1^\mu + p_2^\mu \neq P^\mu$





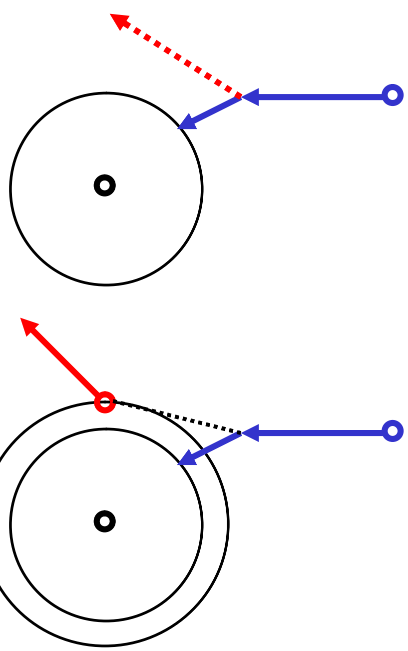
“fragmentation”

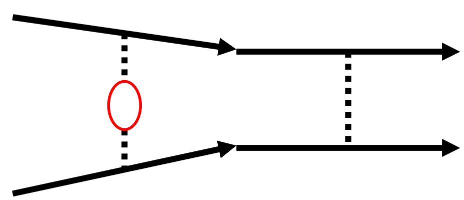


“recombination”

REC

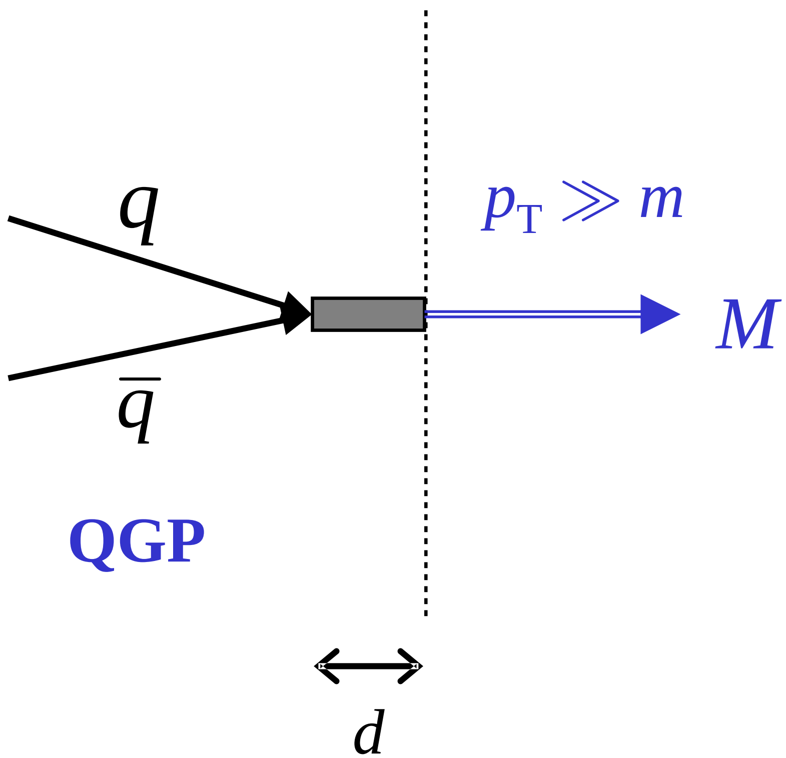
DER





“freeze-out”

Sudden recombination picture



Transition time from QGP into vacuum (in rest frame of produced hadron) is:

$$\tau_f = d/\gamma = d\,\frac{m}{p_T}$$

Allows to ignore complex dynamics in hadronization region; corrections $O(m/p_T)^2$

Not gradual coalescence from dilute system !!!

Tutorial: Non-Relativistic Recombination

Consider system of quarks and antiquarks (no gluons!) of volume V and phase-space distribution $w_a(p) = \langle p,a|\rho|p,a\rangle$.

Quark-antiquark state vs. meson state:

$$\langle x | Q, p_1 p_2 \rangle = V^{-1} e^{i(p_1 \cdot x_1 + p_2 \cdot x_2)}$$
$$\langle x | M, P \rangle = V^{-1} e^{iP \cdot R} \varphi_M(r)$$

with

$$R = \tfrac{1}{2}(x_1 + x_2)$$
$$r = x_1 - x_2$$

Probability for finding a meson with P and $q = (p_1 - p_2)/2$:

$$\left| \langle Q, p_1 p_2 | M, P \rangle \right|^2 = \frac{(2\pi)^3}{V^2} \delta^3(P - p_1 - p_2) \left| \hat{\varphi}_M(q) \right|^2$$

Number of produced mesons:

$$N_M = V \int \frac{d^3P}{(2\pi)^3} \left| \langle M,P | \rho | M,P \rangle \right|^2$$
$$= \sum_{ab} C_M^{ab} V^3 \int \frac{d^3P}{(2\pi)^3} \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} w_a(p_1) w_b(p_1) \left| \langle Q, p_1 p_2 | M,P \rangle \right|^2$$

Tutorial – page 2

The meson spectrum is given by:

$$\frac{dN_M}{d^3P} = \sum_{ab} C_M^{ab} \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} w_a\left(\tfrac{1}{2}P + q\right) w_b\left(\tfrac{1}{2}P - q\right) \left|\hat{\varphi}_M(q)\right|^2$$

Consider case $P \gg q$, where q is of order Λ_M, and expand (for $w_a = w_b$):

$$w\left(\tfrac{1}{2}P + q\right) w\left(\tfrac{1}{2}P - q\right) = w\left(\tfrac{1}{2}P\right)^2 + \sum_{ij} q_i q_j \left[w\left(\tfrac{1}{2}P\right) \partial_i \partial_j w\left(\tfrac{1}{2}P\right) - \partial_i w\left(\tfrac{1}{2}P\right) \partial_j w\left(\tfrac{1}{2}P\right) \right] + \cdots$$

Using only lowest order term:

$$\frac{dN_M}{d^3P} = C_M \frac{V}{(2\pi)^3} \left(w\left(\tfrac{1}{2}P\right)\right)^2 \int \frac{d^3q}{(2\pi)^3} \left|\hat{\varphi}_M(q)\right|^2 = C_M \frac{V}{(2\pi)^3} \left(w\left(\tfrac{1}{2}P\right)\right)^2$$

Corrections are of order $\Lambda_M{}^2 \partial w / P w = \Lambda_M{}^2 / P T$ for thermal quarks.

Tutorial – page 3

The same for baryons:

$$\left|\left\langle Q, p_1 p_2 p_3 \middle| B, P \right\rangle\right|^2 = \frac{(2\pi)^3}{V^2}\delta^3(P - p_1 - p_2 - p_3)\left|\hat{\varphi}_B(q,s)\right|^2$$

where q, s are conjugate to internal coordinates

$$r = \tfrac{1}{2}(x_1 + x_2) - x_3 \quad \text{and} \quad r' = x_1 - x_2$$

The baryon spectrum is then:

$$\frac{dN_B}{d^3P} = \sum_{abc} C_M^{abc} \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \frac{d^3s}{(2\pi)^3} w_a\left(\tfrac{1}{3}P + \tfrac{1}{2}q + s\right) w_b\left(\tfrac{1}{3}P + \tfrac{1}{2}q - s\right) w_c\left(\tfrac{1}{3}P - q\right)\left|\hat{\varphi}_B(q,s)\right|^2$$

$$\approx C_B \frac{V}{(2\pi)^3}\left(w\left(\tfrac{1}{3}P\right)\right)^3\left[1 - O(\Lambda_B{}^2 / PT)\right]$$

Tutorial - page 4

For a thermal Boltzmann distribution $w(p) = \exp\left(-E(p)/T\right)$

we get

$$\left(w\left(\tfrac{1}{2}P\right)\right)^2 = \exp\left[-2E\left(\tfrac{1}{2}P\right)/T\right] = \exp\left(-E_M/T\right)$$

$$\left(w\left(\tfrac{1}{3}P\right)\right)^3 = \exp\left[-3E\left(\tfrac{1}{3}P\right)/T\right] = \exp\left(-E_B/T\right)$$

and therefore:

$$\frac{dN_B(E)}{dN_M(E)} = \frac{C_B}{C_M}\left[1 + O(\Lambda^2/PT)\right]$$

Wigner function formulation

General formulation relies on Wigner functions:

$$\left\langle r_1 - \tfrac{1}{2}r_1', r_2 - \tfrac{1}{2}r_2' \middle| \rho \middle| r_1 + \tfrac{1}{2}r_1', r_2 + \tfrac{1}{2}r_2' \right\rangle = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} e^{-p_1\cdot r - p_2\cdot r_2} W_{ab}(r_1, r_2; p_1, p_2)$$

Meson number becomes:

$$N_M = \sum_{ab} \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} \int d^3R\, d^3r\, d^3r'\; W_{ab}\left(R + \tfrac{1}{2}r, R - \tfrac{1}{2}r; P + \tfrac{1}{2}q, P - \tfrac{1}{2}q\right)$$
$$\times e^{iq\cdot r'} \varphi_M\left(r + \tfrac{1}{2}r'\right) \varphi_M^*\left(r - \tfrac{1}{2}r'\right)$$

Relativistic generalization (u^μ = time-like normal of volume):

$$d^3P\, d^3R \quad \to \quad \frac{P\cdot u}{E} d^3P\, d\Sigma$$

Relativistic formulation

Relativistic formulation using hadron light-cone frame ($P = P_{\|}$):

$$d^3k = \frac{k^0}{k_+} dk^+ d^2k_\perp \quad \text{with} \quad k^+ = \tfrac{1}{\sqrt{2}}\left(k^0 + k_{\|}\right) \quad \text{and} \quad k^+ = xP^+$$

$$E\frac{dN_{\mathrm{M}}}{d^3P} = \int d\Sigma \frac{P\cdot u}{(2\pi)^3} \sum_{\alpha,\beta} \int dx\, w_\alpha(R, xP^+)\overline{w}_\beta(R,(1-x)P^+)\left|\bar{\phi}_{\mathrm{M}}(x)\right|^2$$

$$E\frac{dN_{\mathrm{B}}}{d^3p} = \int d\Sigma \frac{P\cdot u}{(2\pi)^3} \sum_{\alpha,\beta,\gamma} \int dx dx'\, w_\alpha(R, xP^+) w_\beta(R, x'P^+) w_\gamma(R,(1-x-x')P^+)\left|\bar{\phi}_{\mathrm{B}}(x,x')\right|^2$$

For a thermal distribution, $w(r,p) \sim \exp(-p\cdot \mathrm{v}/T)$

the hadron wavefunctions can be integrated out, eliminating the model dependence of predictions. This is true even if higher Fock space states are included!

Beyond the lowest Fock state

$$\left|M;Q^2\right\rangle = \int_0^1 dx_a\,dx_b\,\delta(x_a+x_b-1)\,\phi_1(x_a,x_b;Q^2)\left|q(x_a)\bar{q}(x_b)\right\rangle$$

$$+\int_0^1 dx_a\,dx_b\,dx_c\,\delta(x_a+x_b+x_c-1)\,\phi_2(x_a,x_b,x_c;Q^2)\left|q(x_a)\bar{q}(x_b)g(x_c)\right\rangle + \quad \cdots$$

$$W_{q\bar{q}} = \int_0^1 dx_a\,dx_b\,\delta(x_a+x_b-1)\left|\phi_1(x_a,x_b)\right|^2\left\langle q(x_a)\bar{q}(x_b)\right|\rho\left|q(x_a)\bar{q}(x_b)\right\rangle$$

$$= \int_0^1 dx_a\,dx_b\,\delta(x_a+x_b-1)\left|\phi_1(x_a,x_b)\right|^2 w_q(x_a)w_{\bar{q}}(x_a) = e^{-P/T}\int_0^1 dx_a\left|\phi_1(x_a,1-x_a)\right|^2$$

$$e^{-x_aP/T}\,e^{-x_bP/T} = e^{-(x_a+x_b)P/T}$$

For thermal medium

$$W_{q\bar{q}g} = \int_0^1 dx_a\,dx_b\,dx_c\,\delta(x_a+x_b+x_c-1)\left|\phi_2(x_a,x_b,x_c)\right|^2 w_q(x_a)w_{\bar{q}}(x_a)w_g(x_c)$$

$$= e^{-P/T}\int_0^1 dx_a\,dx_b\left|\phi_2(x_a,x_b,1-x_a-x_b)\right|^2$$

$$e^{-x_aP/T}\,e^{-x_bP/T}\,e^{-x_cP/T} = e^{-(x_a+x_b+x_c)P/T}$$

$$W_M = W_{q\bar{q}} + W_{q\bar{q}g} + \cdots = e^{-P/T}\left[\int_0^1 dx_a\left|\phi_1(x_a,1-x_b)\right|^2 + \int_0^1 dx_a\,dx_b\left|\phi_2(x_a,x_b,1-x_a-x_b)\right|^2 + \cdots\right] = e^{-P/T}$$

Statistical model vs. recombination

In the stat. model, the hadron distribution at freeze-out is given by:

$$E\frac{d^3N_i}{d^3P} = \int_\sigma f_i\left(P\cdot u\right) P^\lambda d\sigma_\lambda \quad \text{with}$$

$$f_i\left(P\cdot u\right) = \frac{g_i}{\left(2\pi\right)^3}\left(\exp\left[\left(P\cdot u - \mu_B B_i - \mu_s S_i - \mu_I I_i\right)/T \pm 1\right]\right)^{-1}$$

For $p_t \rightarrow \infty$, hadron ratios in SM are identical to those in recombination!
(only determined by hadron degeneracy factors & chem. pot.)

- ➢ recombination provides microscopic basis for apparent chemical equilibrium among hadrons at large p_t

BUT: Elliptic flow pattern is approximately additive in valence quarks, reflecting partonic, rather than hadronic origin of flow.

Recombination vs. Fragmentation

Recombination: $$E\frac{dN_{\mathrm{M}}}{d^3P}=\int d\Sigma\,\frac{P\cdot u}{(2\pi)^3}\sum_{\alpha,\beta}\int dx\, w_\alpha(R,xP^+)\overline{w}_\beta(R,(1-x)P^+)\left|\overline{\phi}_{\mathrm{M}}(x)\right|^2$$

Fragmentation: $$E\frac{dN_{\mathrm{h}}}{d^3P}=\int d\sigma\,\frac{P\cdot u}{(2\pi)^3}\int_0^1\frac{dz}{z^3}\sum_\alpha w_\alpha(r,\tfrac{1}{z}P)D_{\alpha\to\mathrm{h}}(z)$$

Recombination… $$w_\alpha(r,xP^+)\overline{w}_\beta(r,(1-x)P^+)=\exp\left(-P\cdot\mathrm{v}/T\right)$$ Meson

$$w_\alpha(r,xP^+)w_\beta(r,x'P^+)w_\gamma(r,(1-x-x')P^+)=\exp\left(-P\cdot\mathrm{v}/T\right)$$ Baryon

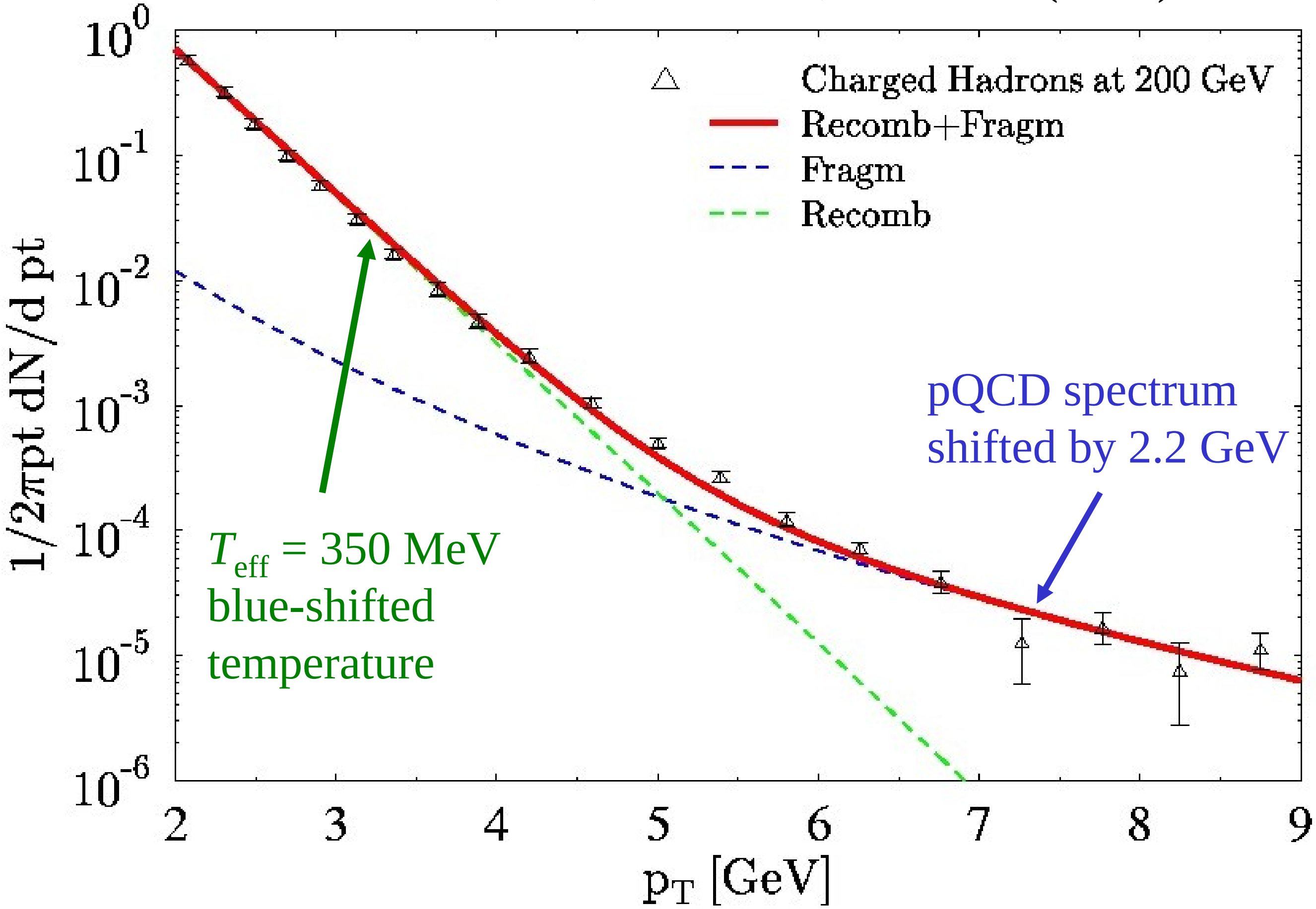
…always wins over fragmentation for an exponential spectrum ($z<1$):

$$\exp(-P\cdot\mathrm{v}/T) \quad > \quad \exp(-P\cdot\mathrm{v}/zT)$$

… but loses at large p_T, where the spectrum is a power law ~ $(p_T)^{-b}$

Model fit to hadron spectrum

R.J. Fries, BM, C. Nonaka, S.A. Bass (PRL)



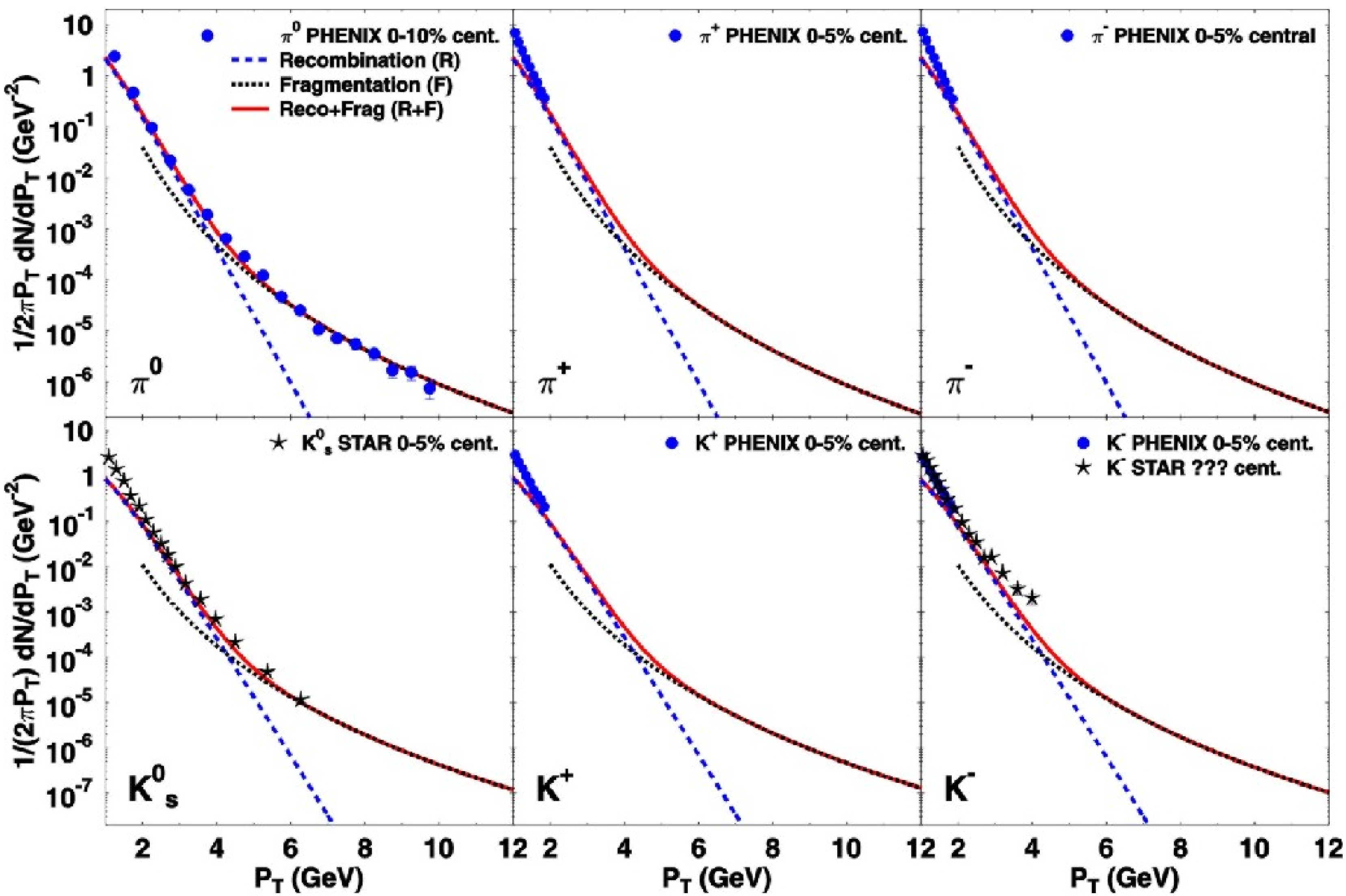
Corresponds to
$\eta = 0.6$!!!

Recall:

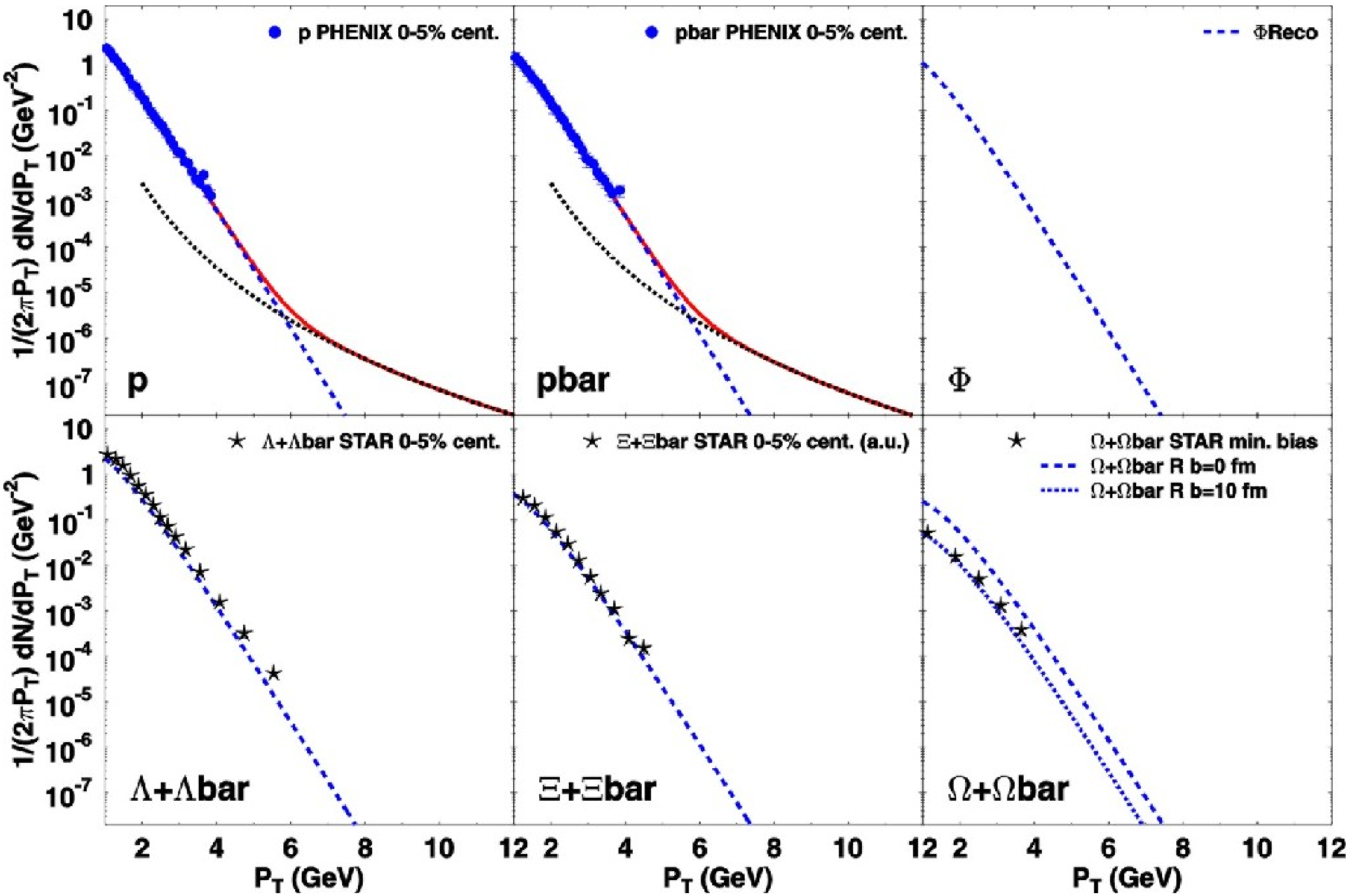
$\eta_G \approx 0.5 \ln(\ldots)$
$\eta_Q \approx 0.25 \ln(\ldots)$

Hadron Spectra I

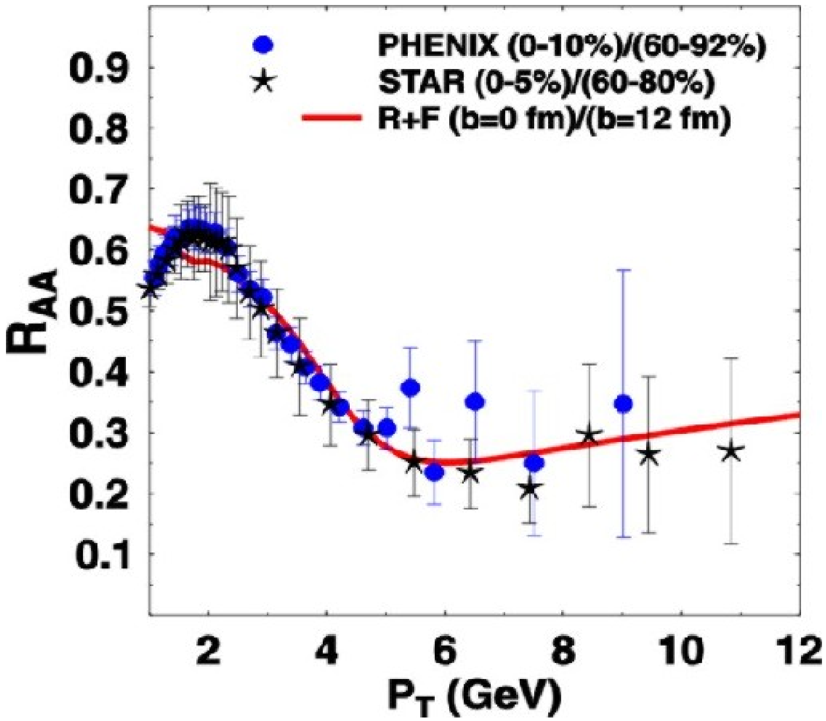
For more details – see S.A. Bass' talk tomorrow

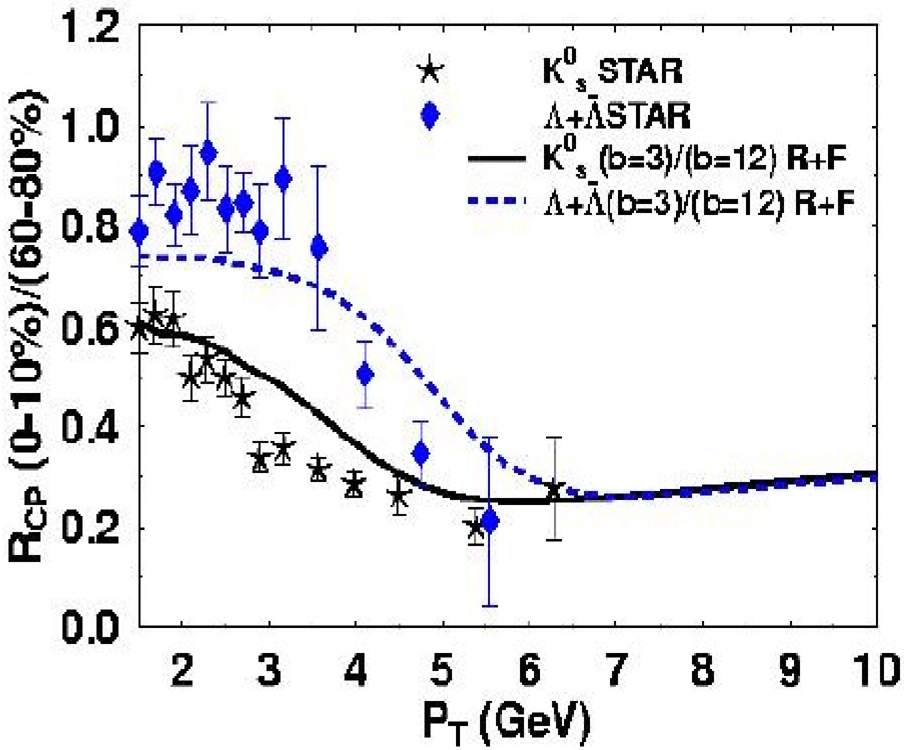


Hadron Spectra II



Hadron dependence of high-p_t suppression

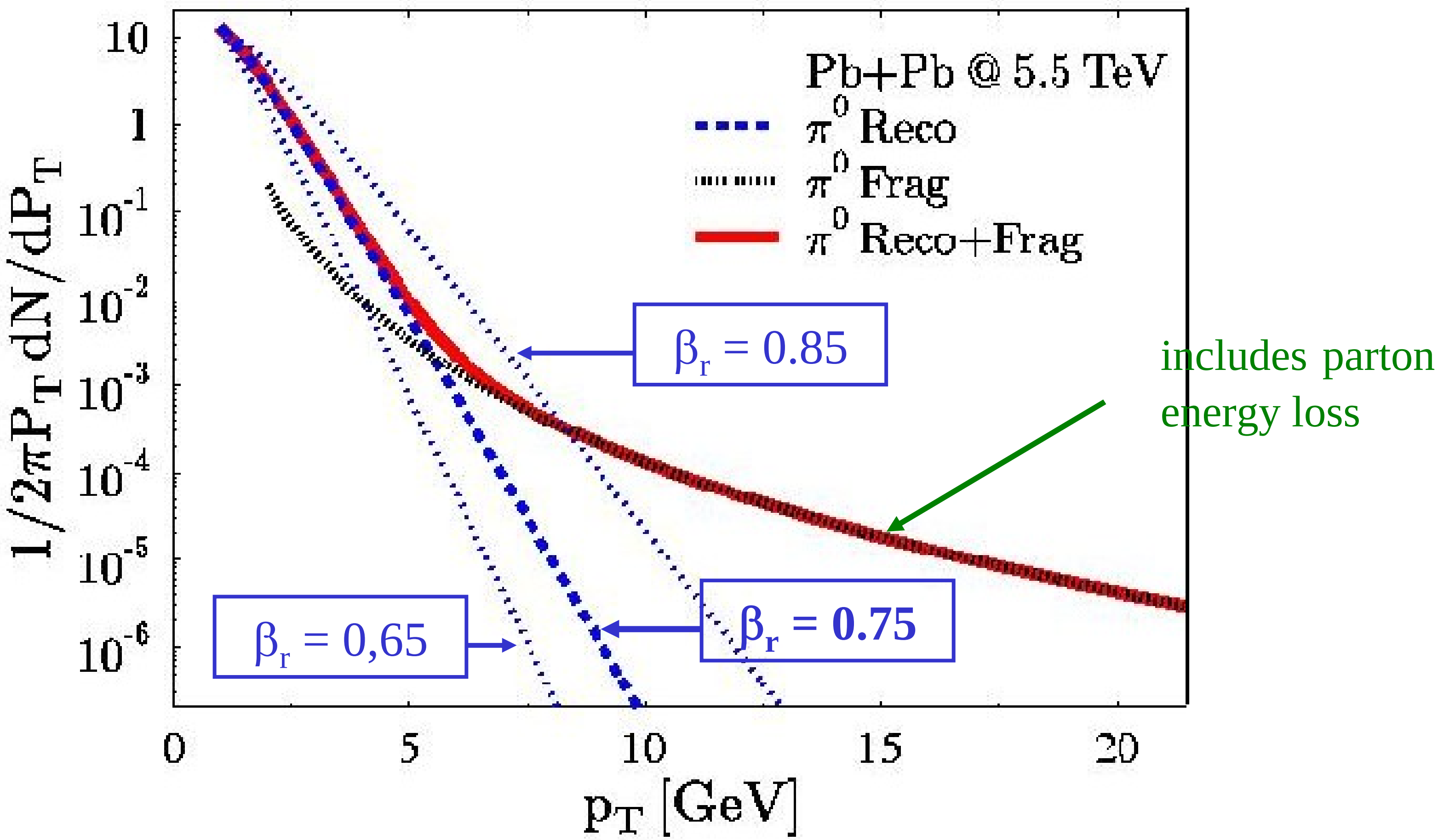




- R+F model describes different R_{AA} behavior of protons and pions
- Jet-quenching becomes universal in the fragmentation region

Hadron production at the LHC

R.J. Fries



Conclusions (1)

- Evidence for dominance of hadronization by quark recombination from a thermal, deconfined phase comes from:
  - Large baryon/meson ratios at moderately large p_T;
  - Compatibility of measured abundances with statistical model predictions at rather large p_T;
  - Collective radial flow still visible at large p_T.
- Φ-meson is an excellent test case ~~(if not from $KK \to \Phi$)~~.

Parton Number Scaling of Elliptic Flow

In the recombination regime, meson and baryon v_2 can be obtained from the parton v_2 (using $x_i = 1/n$):

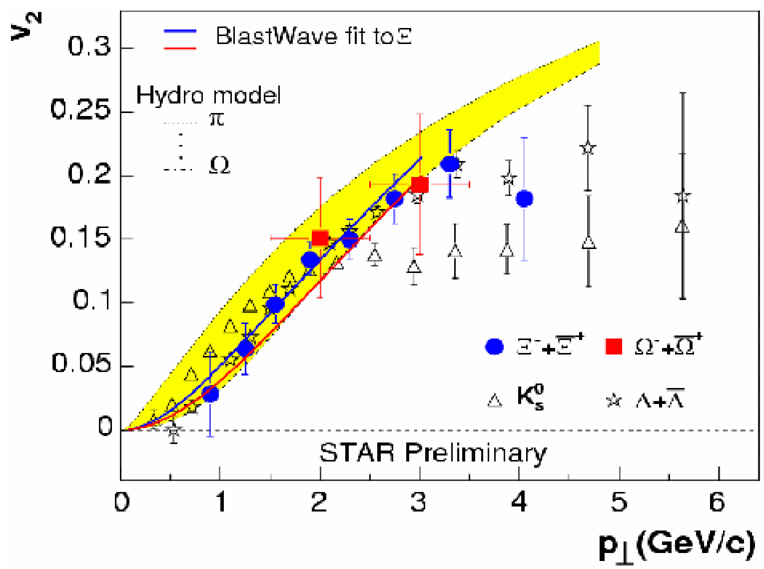
$$v_2^M(p_t) = \frac{2v_2^p\left(\frac{p_t}{2}\right)}{1+2\left(v_2^p\left(\frac{p_t}{2}\right)\right)^2} \quad \text{and} \quad v_2^B(p_t) = \frac{3v_2^p\left(\frac{p_t}{3}\right)+3\left(v_2^p\left(\frac{p_t}{3}\right)\right)^3}{1+6\left(v_2^p\left(\frac{p_t}{3}\right)\right)^2}$$

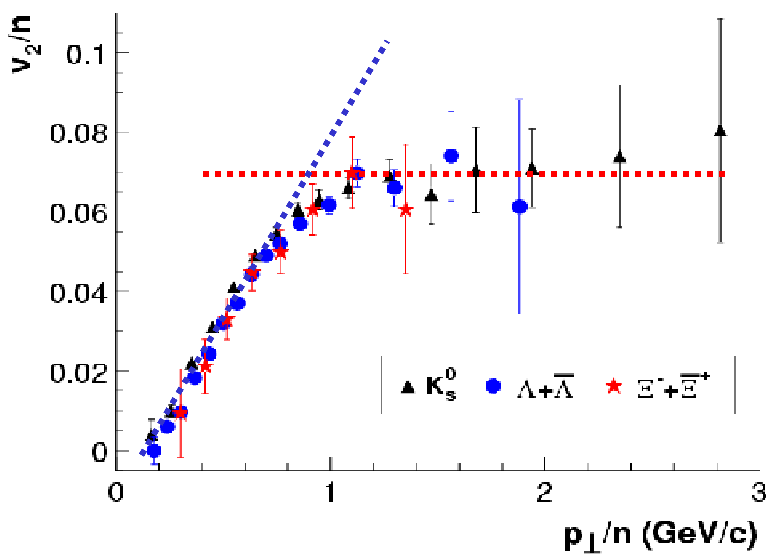
Neglecting quadratic and cubic terms, a simple scaling law holds:

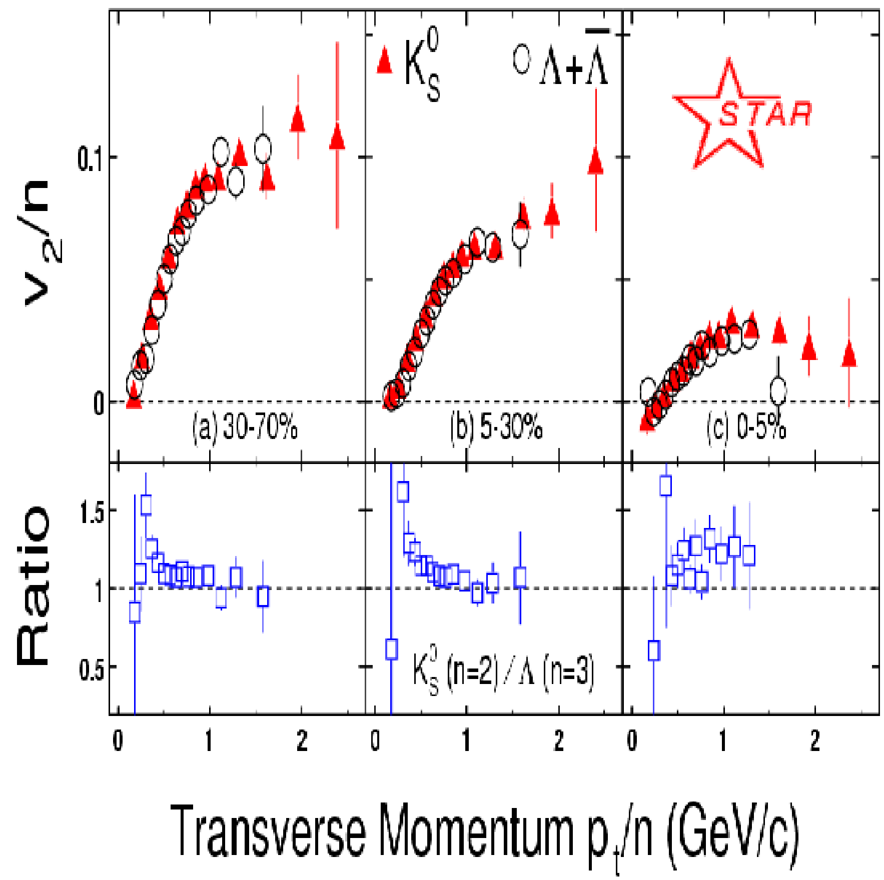
$$v_2^M(p_t) = 2v_2^p\left(\frac{p_t}{2}\right) \quad \text{and} \quad v_2^B(p_t) = 3v_2^p\left(\frac{p_t}{3}\right)$$

Originally poposed by S. Voloshin

Hadron v_2 reflects quark flow !







Higher Fock states don't spoil the fun

$$\phi_1^{(M)}(x_a, x_b) \sim x_a x_b$$

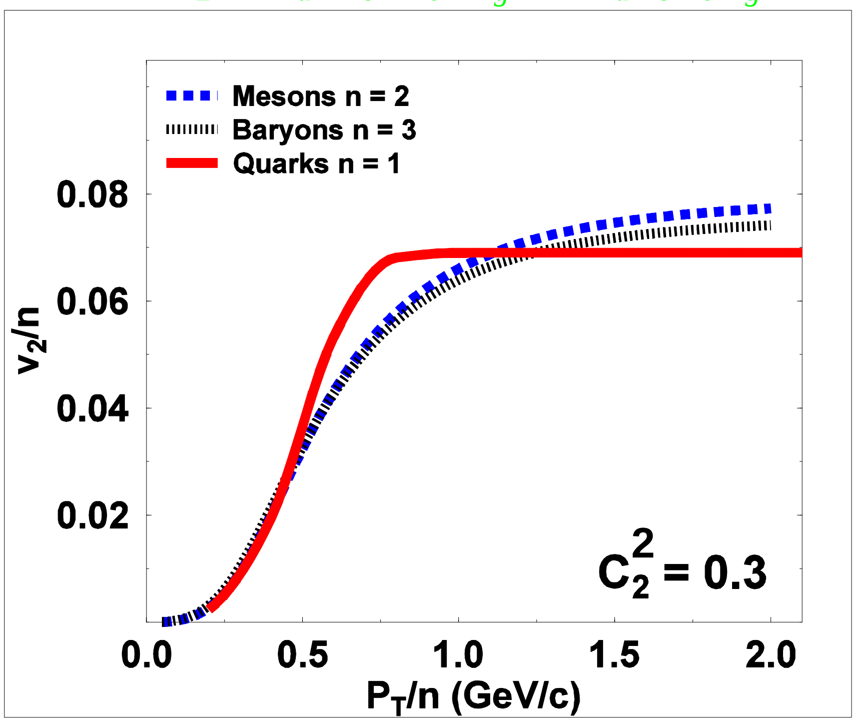
$$\phi_2^{(M)}(x_a, x_b, x_g) \sim x_a x_b x_g^2$$

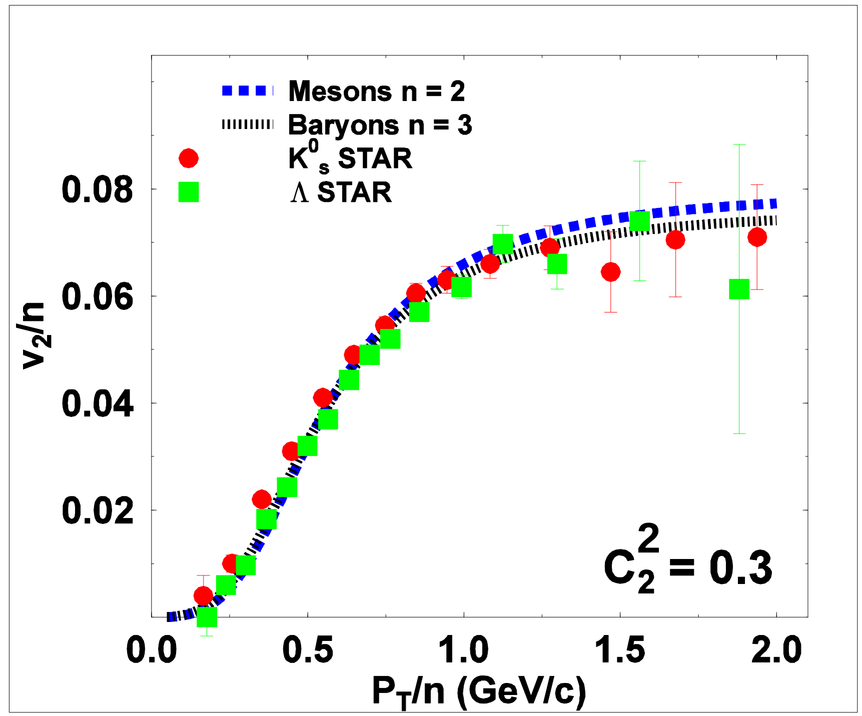
$$\phi_1^{(B)}(x_a, x_b, x_c) \sim x_a x_b x_c$$

$$\phi_2^{(B)}(x_a, x_b, x_c, x_g) \sim x_a x_b x_c x_g^2$$

$$|M\rangle = C_1 |q\bar{q}\rangle + C_2 |q\bar{q}g\rangle$$

$$|B\rangle = C_1 |qqq\rangle + C_2 |qqqg\rangle$$





Conclusions (2)

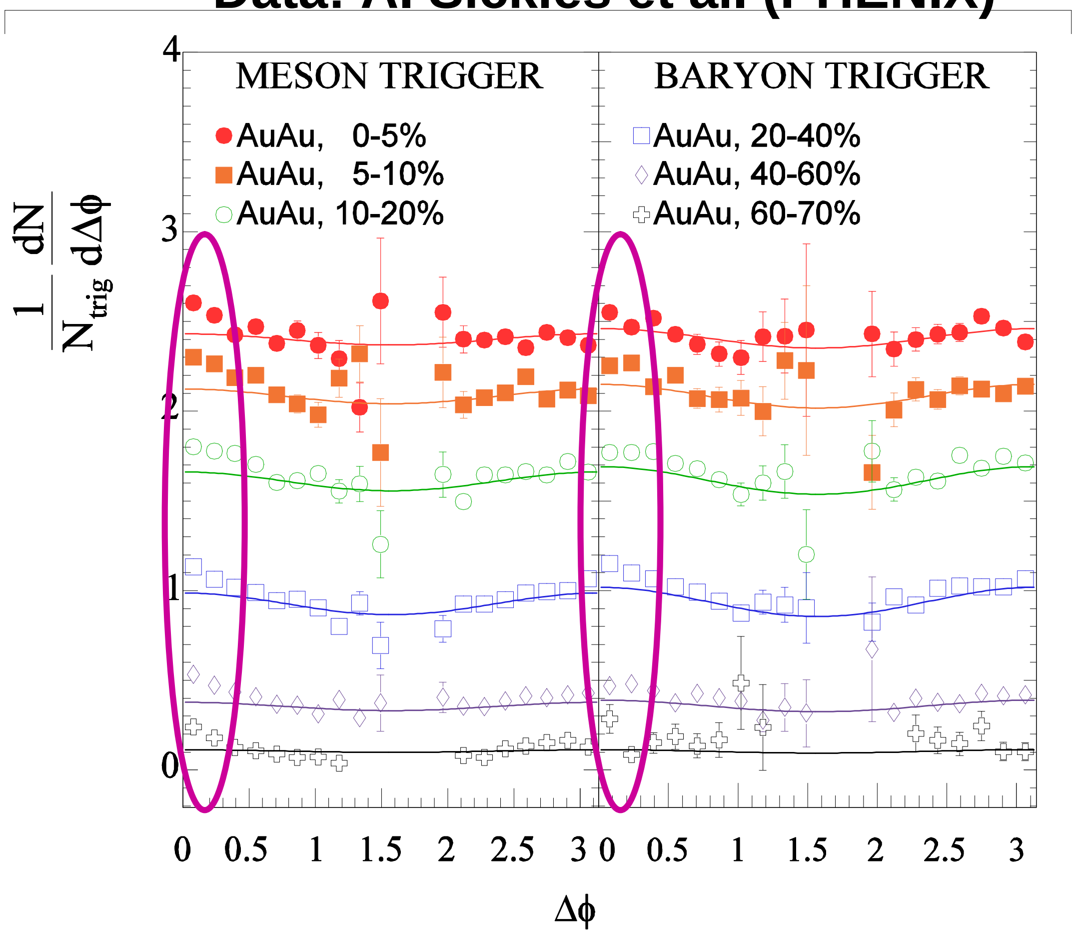
- Recombination model works nicely for $v_2(p)$:
  - $v_2(p_T)$ curves for different hadrons collapse to *universal* curve for constituent quarks;
  - Saturation value of v_2 for large p_T is *universal* for quarks and agrees with expectations from anisotropic energy loss;
  - Vector mesons (Φ, K^*) permit test for influence of mass versus constituent number (but note the *effects of hadronic rescattering on resonances*!);
  - Higher Fock space components can be accommodated.

Enough of the Successes…

Give us some Challenges!

Dihadron correlations

Data: A. Sickles et al. (PHENIX)



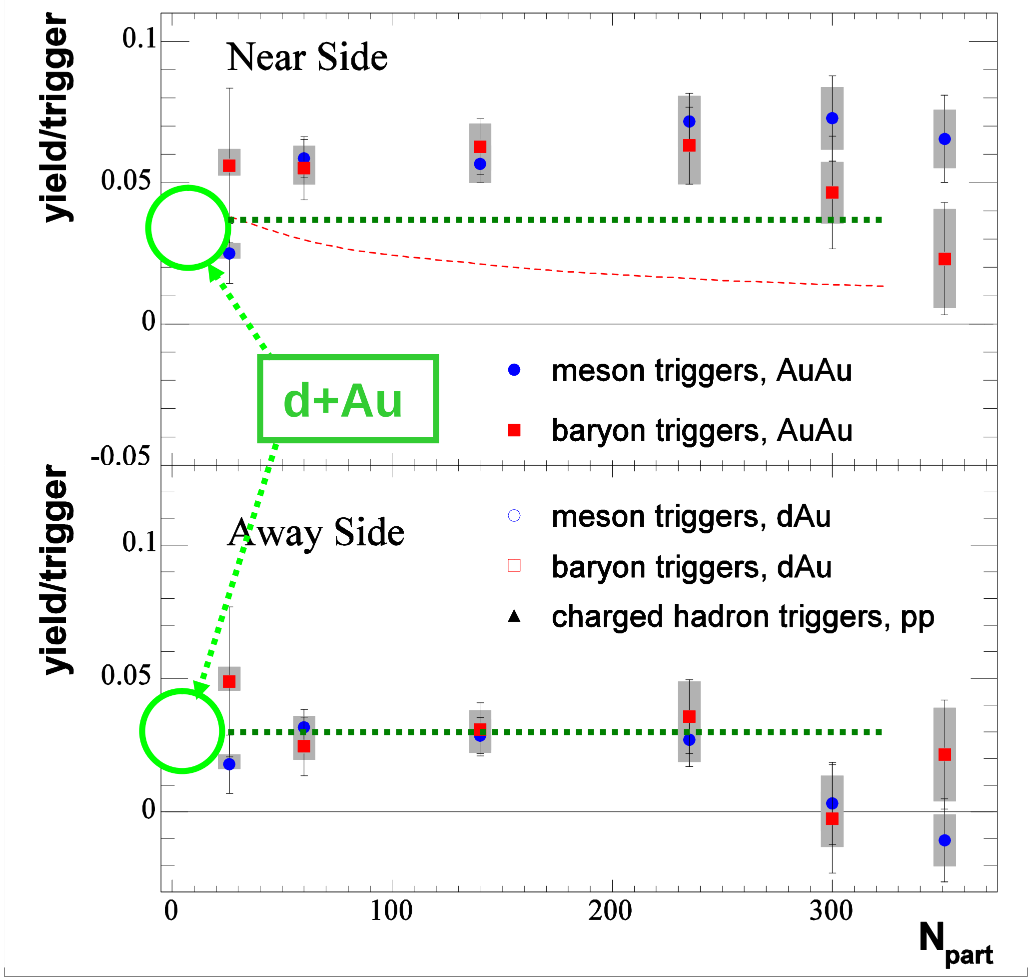
Hadrons created by reco from a thermal medium should not be correlated.

But jet-like correlations between hadrons persist in the momentum range ($p_T \leq 4$ GeV/c) where recombination is thought to dominate!

(STAR + PHENIX data)

Hadron-hadron correlations

A. Sickles et al. (PHENIX)



Near-side dihadron correlations are larger than in d+Au !!!

Far-side correlations disappear for central collisions.

Sources of correlations

- Standard fragmentation
- Fragmentation followed by recombination with medium particles
- Recombination from (incompletely) thermalized, correlated medium

- But how to explain the baryon excess?
- "Soft-hard" recombination (Hwa & Yang). Requires microscopic fragmentation picture
- Requires assumptions about two-body correlations (Fries et al.)

How serious is this?

- Original recombination model is based on the assumption of a one-body quark density. Two-hadron correlations are determined by *quark correlations*, which are not included in pure thermal model.

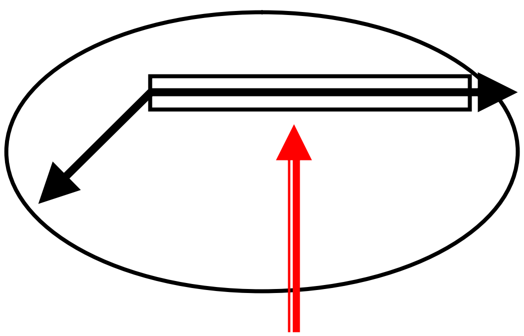
- Two- and multi-quark correlations are a natural result of jet quenching by energy loss of fast partons.

- Incorporation of quark correlations is straightforward, but introduces new parameters: $C(p_1, p_2)$.

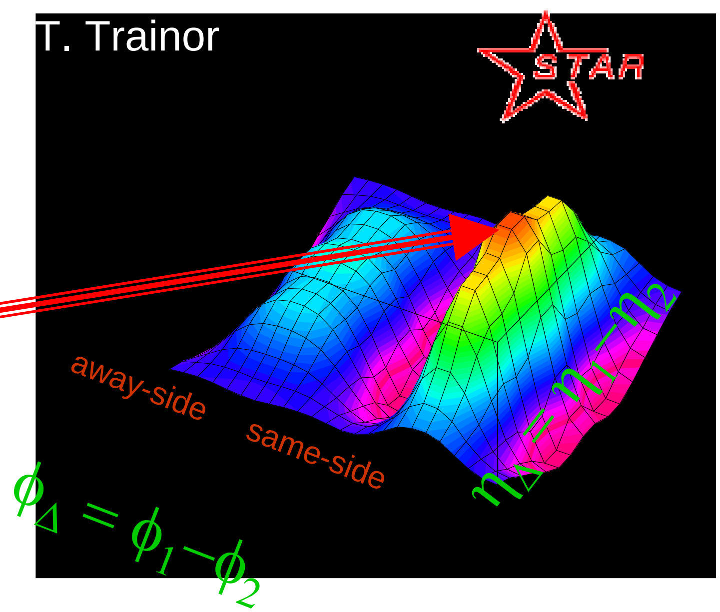
Diparton correlations

A plausible explanation?

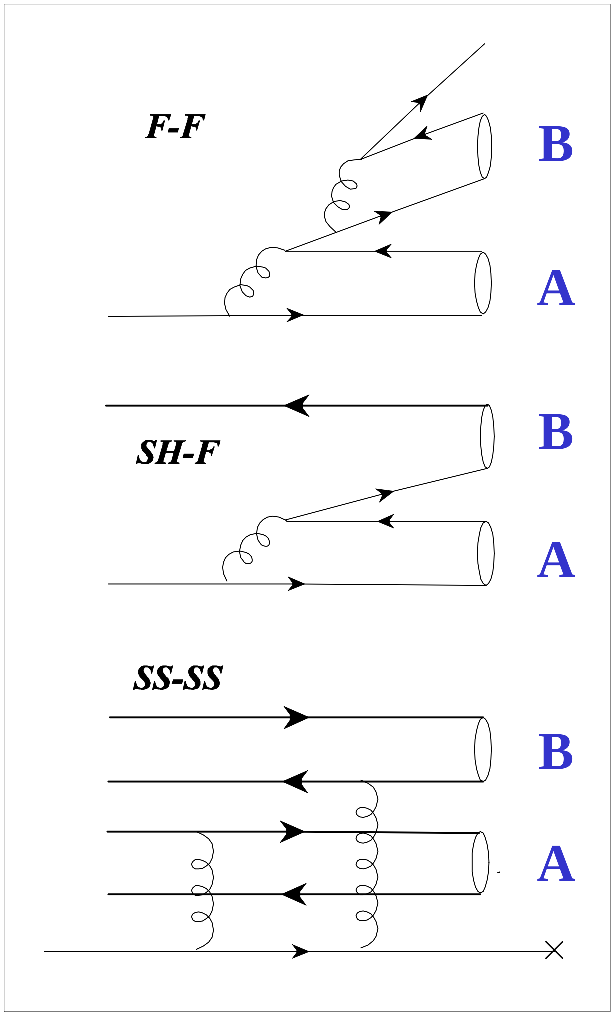
- **Parton correlations naturally translate into hadron correlations.**
- **Parton correlations likely to exist even in the "thermal" regime, created as the result of stopping of suprathermal partons.**



Two-point velocity correlations among 1-2 GeV/c hadrons



Dihadron formation mechanisms



$$F-F \quad \sim \int \frac{dz_A}{z_A(1-z_A)} g_a\left(\frac{P_A}{z_A}+\Delta E\right) \times D(z_A) D\left(\frac{z_A P_B}{(1-z_A)P_A}\right)$$

$$SH-F \quad \sim g_a\left(P_A+\tfrac{1}{2}P_B+\Delta E\right) \times D\left(\frac{P_A}{P_A+\frac{1}{2}P_B}\right)\exp\left(-\frac{P_B}{2T_{\rm eff}}\right)$$

$$SS-SS \quad \sim \exp\left(-\frac{P_A+P_B}{T_{\rm eff}}\right)$$

Correlations - formalism

Di-meson production:

$$\frac{dN_{MM}}{d^3P_1 d^3P_2} = \frac{V^2}{(2\pi)^6} \int d^3q_1 d^3q_2 |\phi(q_1)|^2 |\phi(q_2)|^2 W_4\left(\frac{1}{2}P_1 + q_1, \frac{1}{2}P_1 - q_1, \frac{1}{2}P_2 + q_2, \frac{1}{2}P_2 + q_2\right)$$

$$W_n(p_1, \ldots, p_n) = \prod_n w(p_i)\left(1 + \sum_{i<j} C_{qq}\left(p_i, p_j\right)\right)$$

Partons with pairwise correlations

$$\Rightarrow \frac{dN_{MM}}{d^3P_1 d^3P_2} = \frac{V^2}{(2\pi)^6} w^2\left(\frac{1}{2}P_1\right) w^2\left(\frac{1}{2}P_2\right)\left[1 + 2C_0 + 4C_{qq}\left(\frac{1}{2}P_1, \frac{1}{2}P_2\right)\right]$$

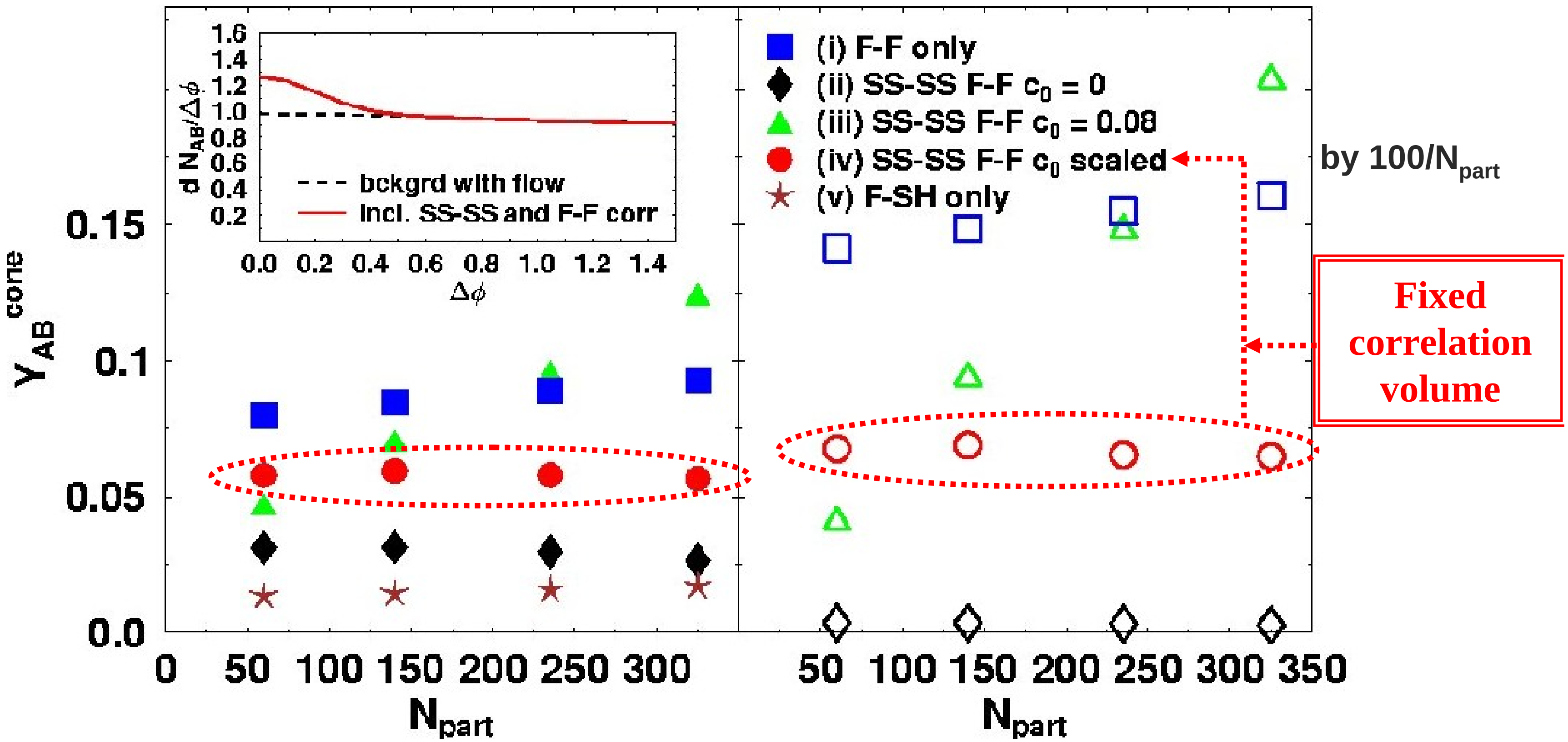
Meson-meson, baryon-baryon, baryon-meson correlations

$$C_{BB} = 9C_{qq}, \quad C_{MB} = 6C_{qq}, \quad C_{MM} = 4C_{qq}$$

First results of model studies are encouraging ?

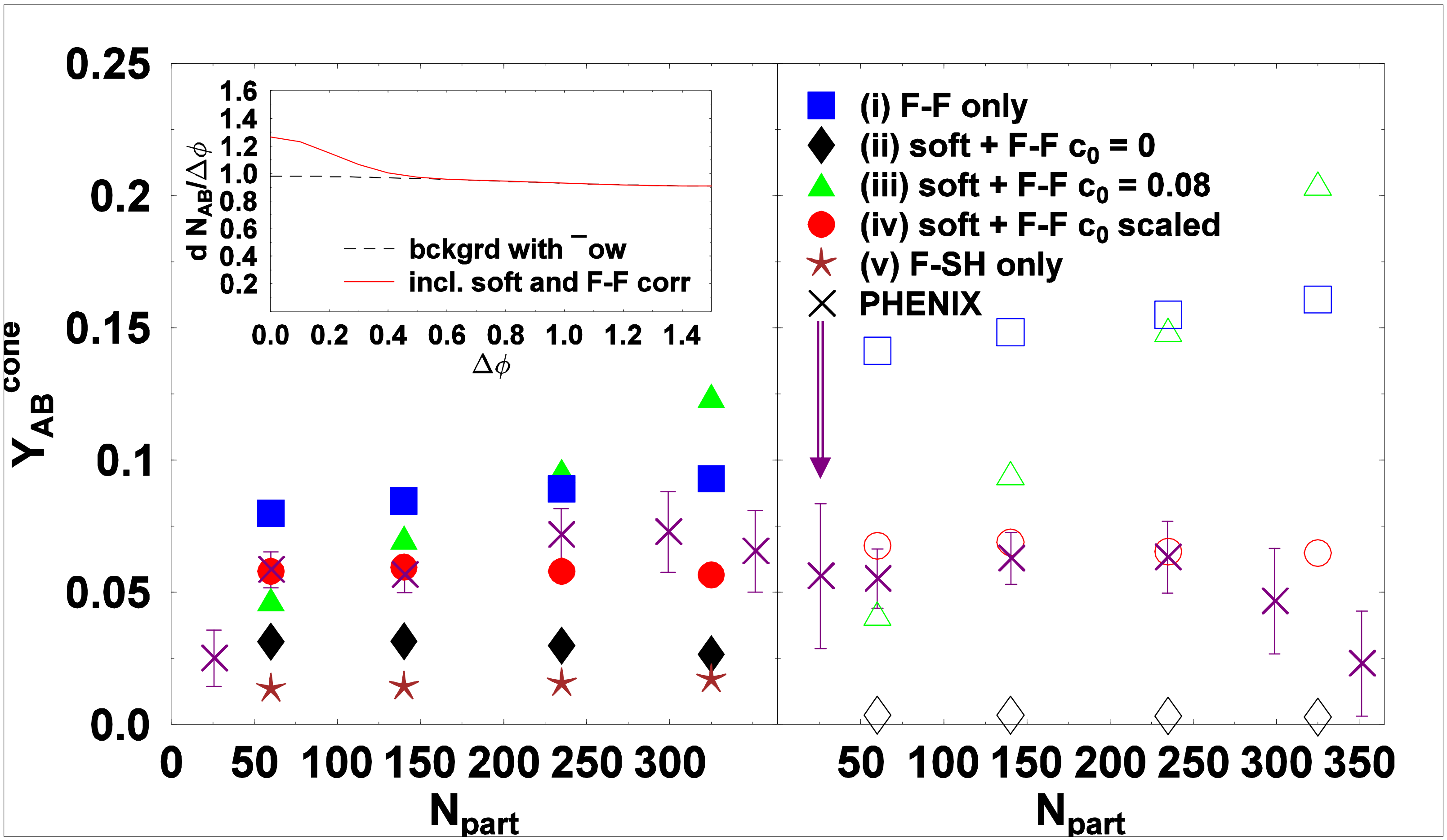
Dihadron correlations - results

Meson triggers **Baryon triggers**



Comparison with Data

R.J. Fries, S.A. Bass, BM, nucl-th/0407102, acct'd in PRL



Conclusions – at last!

Evidence for the formation of a deconfined phase of QCD matter at RHIC:

- ✓ *Hadrons are emitted in **universal** equilibrium abundances;*
- ✓ *Most hadrons are produced by **recombination of quarks**;*
- ✓ *Hadrons show evidence of **collective flow*** (v_0 *and* v_2);
- ✓ *Flow pattern* (v_2) *is not universal for hadrons, but **universal for the (constituent) quarks**.*
- ❖ *Hadron correlations from quasithermal quark correlations.*