Physics with Two Time-like Dimensions

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The problem with 2 times

- While theories with more than 3 space-like dimensions have been quite popular in particle physics (an understatement !), theories with more than one time-like dimension have been shunned, for four reasons:
  - Violations of causality (closed time-like loops);
  - Violations of unitarity;
  - Existence of tachyonic modes;
  - Presence of ghost fields.

- While technically different in nature, these 4 problems are interrelated and possibly may be resolved by a common mechanism.

What this talk is *not* about ...

- One solution (I. Bars, 1997 ff.) is to impose a symplectic gauge constraint on states in D+2 dimensions:

$$X^2 |\Phi(X)\rangle = P^2 |\Phi(X)\rangle = (X \cdot P + P \cdot X)|\Phi(X)\rangle = 0$$

$$\Longrightarrow \qquad |\Phi(X)\rangle \propto \delta(X^2) = \delta(t_1^2 + t_2^2 - x_1^2 - \ldots - x_D^2)$$

- This reduces the dynamics to (D-1)+1 dimensions and eliminates all problems associated with a second time-like dimension, but also renders time effectively 1-dimensional.
- The gauge constraints lead to certain constraints on the (D-1)+1 dimensional field theory, e.g., no θ-term in QCD.

Time-like loops - I

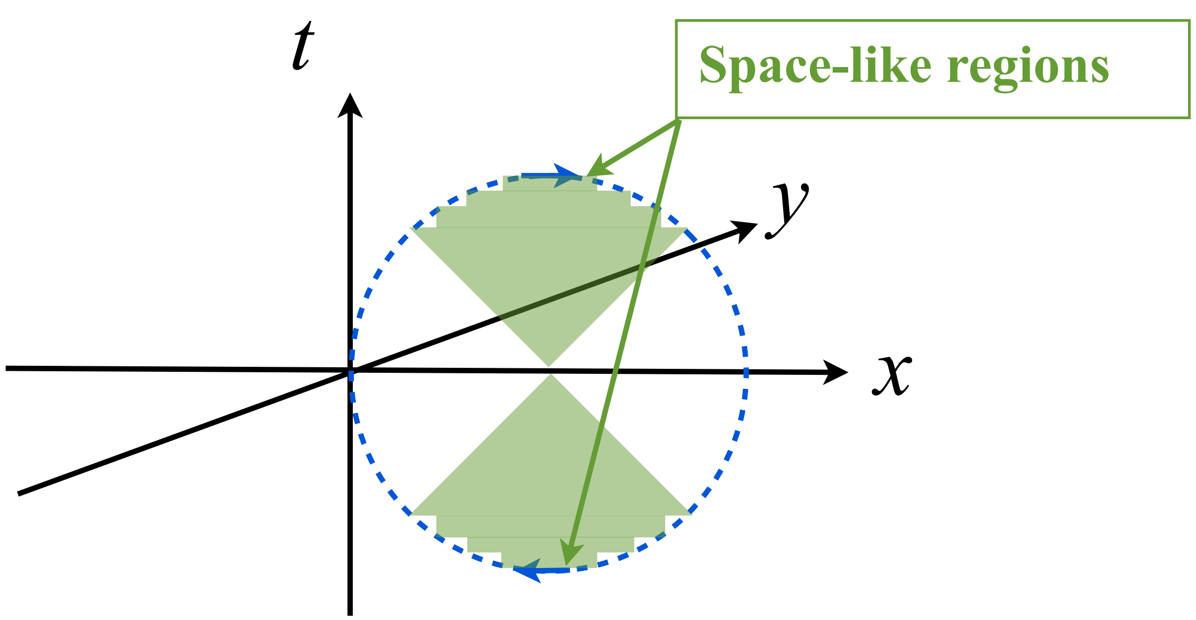
Causality violation originates from the possibility of time-like loops in world lines.

These are impossible in (flat) space-time with a single time dimension, but are common-place in space-times with 2 or more time-like dimensions:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$\begin{aligned} t &= r \sin\alpha \\ x &= r\,(1 - \cos\alpha) \\ &\Downarrow \\ ds^2 = dt^2 - dx^2 &= (\cos 2\alpha)\; r^2 d\alpha^2 \\ &< 0 \qquad \text{for } \frac{\pi}{4} < \alpha < \frac{3\pi}{4} \end{aligned}$$

World-line is partly space-like and thus unphysical !



Time-like loops - II

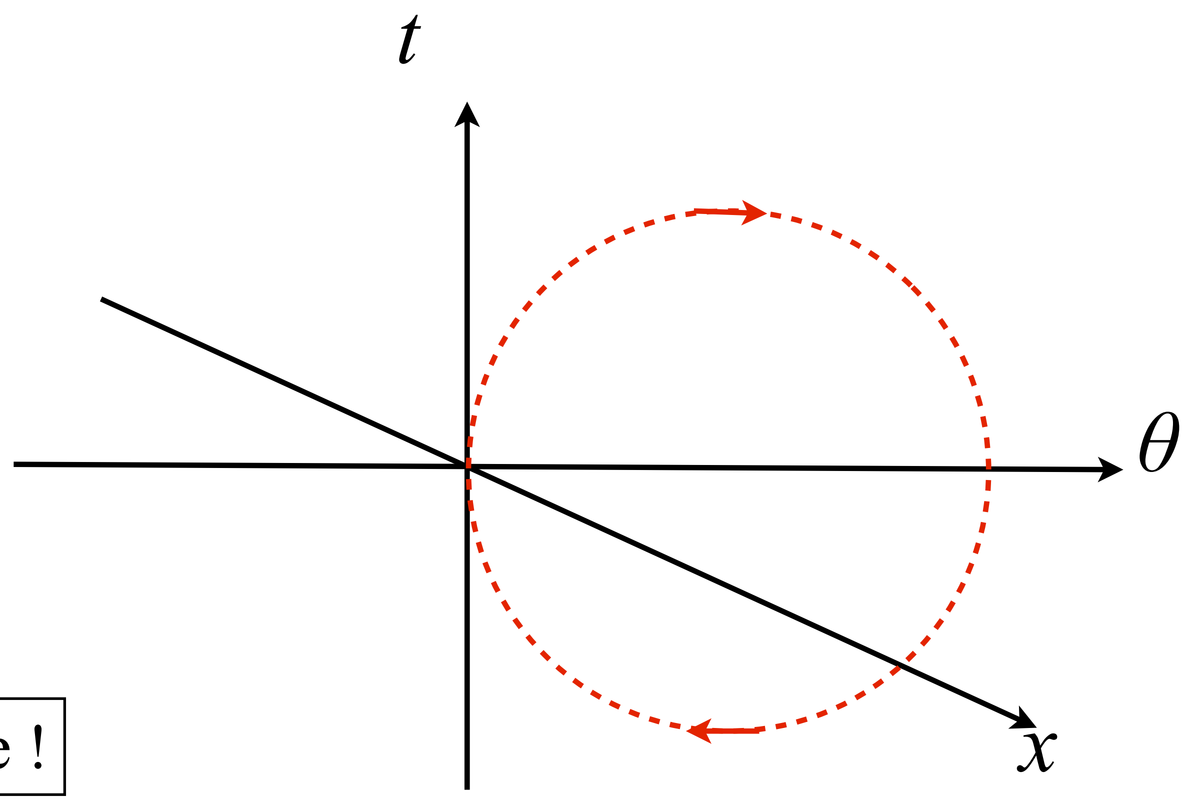
In a space-time with a second time-like dimension we can trade off "motion" in one time dimension against motion in the second time dimension and return to the same moment.

$$\boxed{ds^2 = dt^2 + d\theta^2 - dx^2 - dy^2 - dz^2}$$

$$\begin{aligned} t &= r\sin\alpha \\ \theta &= r\,(1-\cos\alpha) \\ x &= \text{const.} \\ &\Downarrow \end{aligned}$$

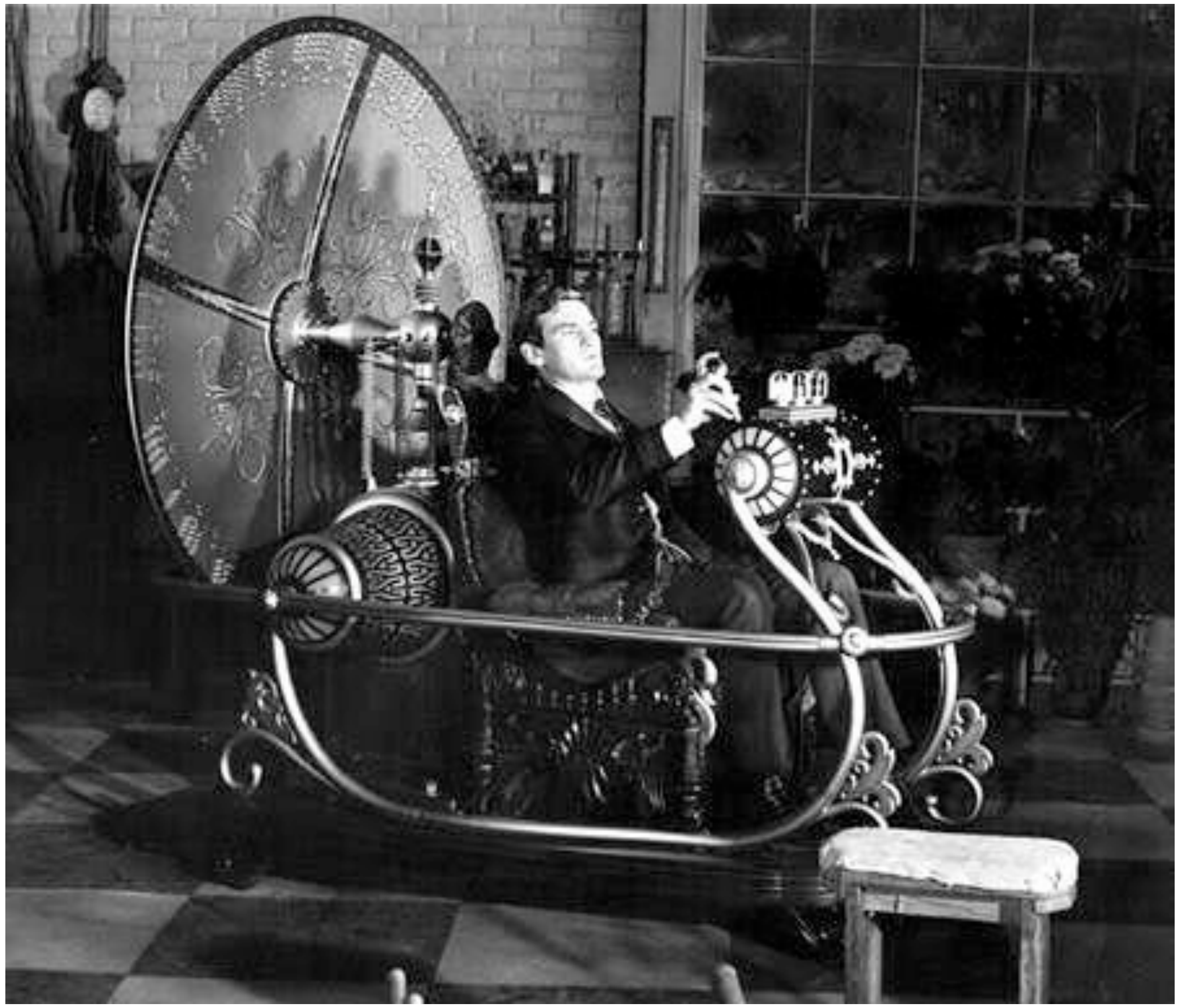
$$ds^2 = dt^2 + d\theta^2 - dx^2 \quad = \quad r^2 d\alpha^2 > 0$$

World-line is everwhere time-like !



The Grandfather Paradox

What's so bad about time-like loops?

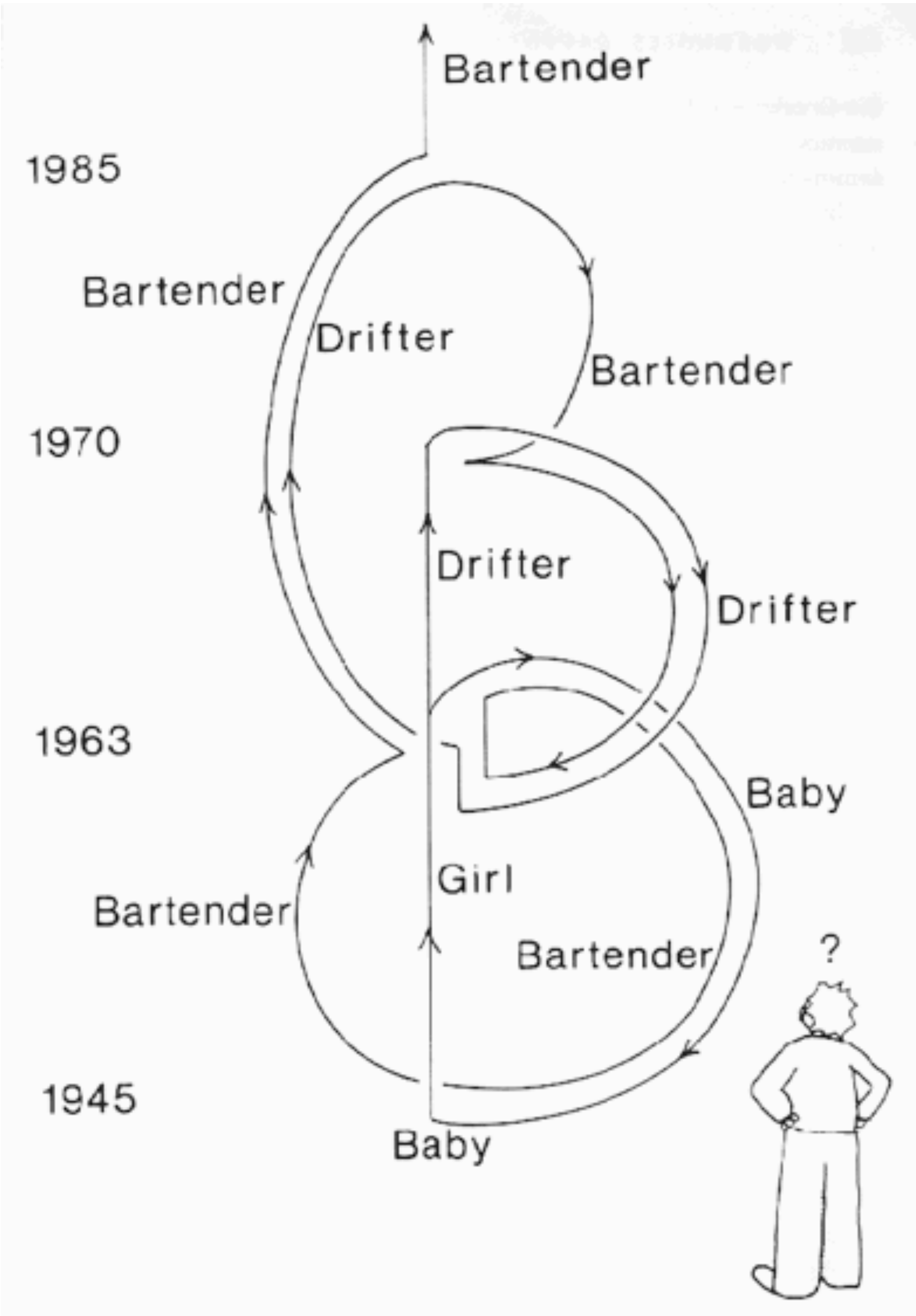


H. G. Wells: *The Time Machine*

The Grandfather Paradox

If you could travel back in time, you could return to a time before your grandfather met your grandmother and shoot to kill him. Then your mother and thus you, the inventor of the time machine, would never be born. You would *not* invent the time machine, could *not* travel back in time, and your grandfather would *not* be shot. And thus, you would be born, and .....

Paradoxes everywhere

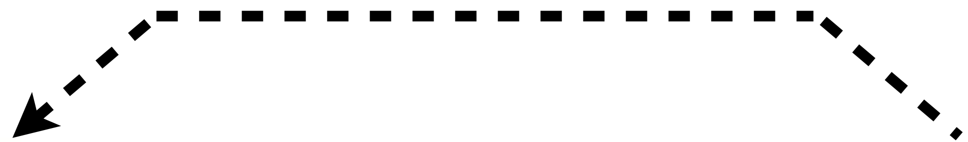


Heinlein (in "*All You Zombies*") imagined a world line that is a convoluted closed loop:

In 1945, a girl is born. In 1963 she has a baby from a stranger; then undergoes sex change surgery. In 1970 "he" is a drifter, who goes back to 1963 to meet himself. In 1985 he is a time traveler, who picks himself up in a bar in 1970, takes himself back to 1963, kidnaps the baby and takes her back to 1945, to start all over again.

The girl is her own mother, father, grand-father, grandmother, son, daughter, etc.

The BiT-Phone Paradox



The Plan

Send a message tomorrow, if and only if no message arrives today.

Today

Tomorrow

This is really, really bad !

Avoiding disaster

- The causality problem arises due to assumption that motion in the second time dimension (θ) is "regular", i.e. simply defined by initial condition. This does not have to be the case. The dependence on θ could be chaotic in the sense that arbitrarily small changes in the initial conditions lead to exponentially growing deviations. Finding a closed time-like loop then becomes exponentially unlikely. So does visiting a specific event in the past.
- The "*Butterfly" Effect*: A butterfly flapping its wings in Africa can create (or avoid) a tornado in America. But we would not assign a *causal* significance to the wing flap, because an exponentially large number of "causes" with comparable significance exist.
- Over long times, chaotic dynamics is best described as micro-canonically *ergodic*, or for a sufficiently large system, as canonically ergodic, i.e. as *thermal*. This suggests that we should consider space-times with two time-like dimensions, in which one dimension (θ) is thermalized.

Hot and cold time

When 2 dimensions are time-like, the *energy* is a 2-component vector:

$$x^\mu = (t, \theta, \mathbf{x}) \qquad \Leftrightarrow \qquad p^\mu = (E_1, E_2, \mathbf{p}) = (\vec{E}, \mathbf{p})$$

The (inverse) temperature is then also a 2-component vector: $\vec{\beta} = (\beta_1, \beta_2)$

If the same temperature vector exists everywhere in space-time, we can *define* the global direction of the temperature vector as the θ direction. The physical time direction is then defined as the (time-like) direction orthogonal to $\vec{\beta}$.

In other words: The dynamics in the additional “hot” time dimension is diffusive; the dynamics in the “cold” physical time dimension is ballistic.

We imagine that $\beta^{-1} = T$ is very large so that the microscopic details behind the thermal dynamics are not visible at time and distance scales accessible today.

If time travel is erratic, you just might get lucky...

...but your chances of returning would be exponentially slim.

Causality revisited

Partial differential equations of the type $$\left(\sum_{i=1}^{q}\frac{\partial^2}{\partial t_i^2}-\sum_{k=1}^{p}\frac{\partial^2}{\partial x_k^2}\right)u(t_i;x_k)=0$$

with $q > 1$ are called ultra-hyperbolic differential equations. The initial value (Cauchy) problem was studied beginning in the 1930's. Main result (F. John - 1938):

If the linear PDE has a global solution for given initial conditions $u(0,t_2,\ldots t_q;x_1,\ldots,x_p)$, this solution is unique.

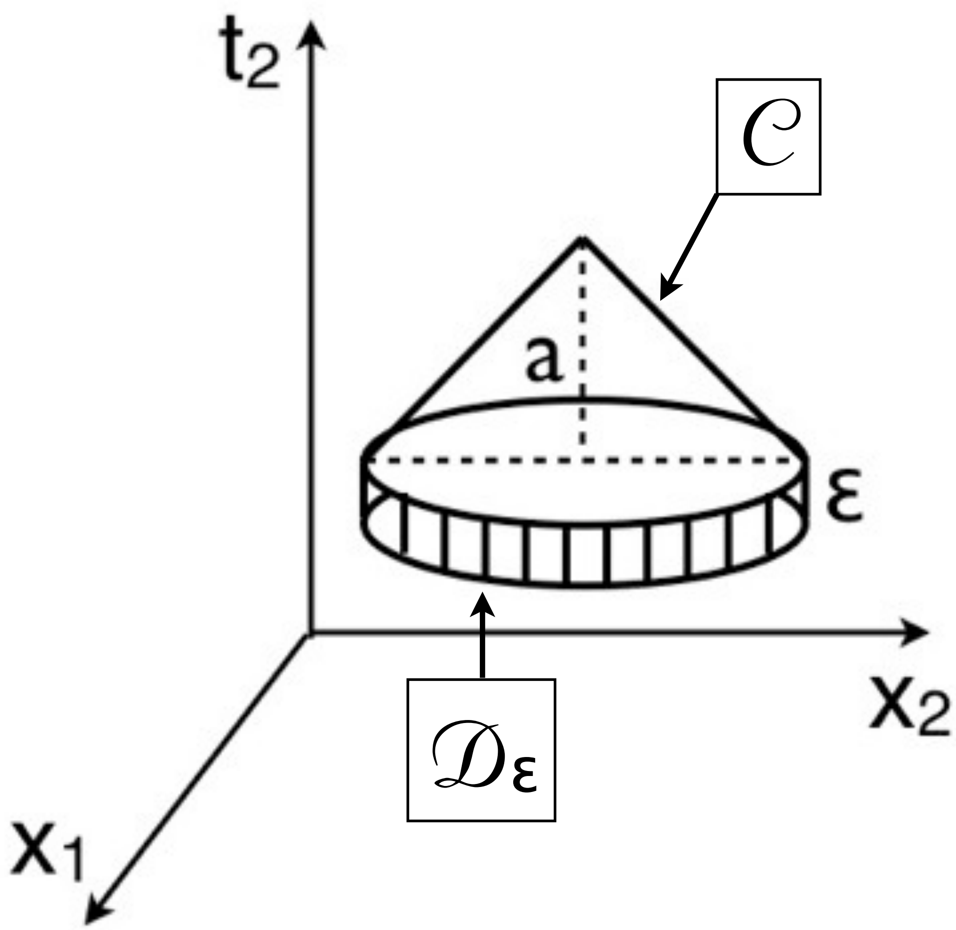
The proof is based on a generalized mean value theorem due to Asgueirsson (1936).

Unfortunately, the proof does not ensure that a global solution exists...

Courant (1962) argued that no admissible solution exists for general initial data!

Craig and Weinstein (2008) gave a non-local condition which ensures the existence of a global solution for linear PDE's with $q = 2$. Unfortunately, the construction only works for linear ultra-hyperbolic PDE's, but it suggests a general resolution.

Courant's argument



Consider the interesting case $q = 2$ (two times). The initial data are given in a region $(t_2; x_1, x_2, \ldots)$.

Asgueirsson's theorem implies that the values of $u(t_2; x_1, x_2, \ldots)$ in the cone $\mathcal{C}$

$$t_1 = 0; \qquad |t_2 - t_2^{(0)}| + |\mathbf{x} - \mathbf{x}^{(0)}| \leq a$$

are fully determined by the values $u(t_2; x_1, x_2, \ldots)$ taken in the infinitesimal disk $\mathcal{D}_\varepsilon$

$$t_1 = 0; \qquad |t_2 - t_2^{(0)}| \leq \varepsilon; \qquad |\mathbf{x} - \mathbf{x}^{(0)}| \leq a$$

This implies that the initial data on $\mathcal{C}$ cannot be arbitrarily chosen. But it also means that the initial value problem is determined by data on the "thickened" two-time hyper-plane $\{ t_1 = 0, |t_2| \leq \varepsilon; x_1, x_2, \ldots \}$.

The C-W "solution"

To understand the origin of the problem, go into Fourier space:

$$(t_1, t_2; \mathbf{x}) \equiv (t, \theta; \mathbf{x}) \qquad \Longleftrightarrow \qquad (\omega, \chi; \mathbf{k})$$

Ultra-hyperbolic PDE implies the dispersion relation $\omega^2(\chi, \mathbf{k}) = \mathbf{k}^2 - \chi^2$

which permits exponentially growing (in time *t*) solutions when |χ| > |***k***|.

If the initial conditions are constrained to $u_0(\chi; \mathbf{k}) = u_0'(\chi; \mathbf{k}) = 0 \quad \text{for } \chi^2 > \mathbf{k}^2$
then a globally admissible solution is given by:

$$u(t, \theta; \mathbf{x}) = \int d\chi \, d\mathbf{k} \left(u_0(\chi; \mathbf{k}) \cos[\omega(\chi, \mathbf{k})t] + \frac{u_0'(\chi; \mathbf{k})}{\omega(\chi, \mathbf{k})} \sin[\omega(\chi, \mathbf{k})t] \right) e^{i\mathbf{k}\cdot\mathbf{x} - i\chi\theta}$$

Problem: If the PDE contains nonlinear terms, Fourier components in the "wrong" domain are automatically generated by the time evolution. But all interacting field theories correspond to nonlinear wave equations! For such a nonlinear theory to be acceptable, the offensive modes must be quenched dynamically, which may happen at nonzero temperature.

Possible theories

- Acceptable theories must be nonlinear theories of a specific kind:
  - At nonzero temperature T all time-like modes must be exponentially quenched.
- An example of such a theory is the strongly coupled SU(N_c) super-Yang-Mills theory at large N_c which has no identifiable propagating time-like modes at all - or more generally, theories that have been called “un-particle” theories. The only weakly damped, quasiparticle type mode of the strongly coupled super-YM theory is the (space-like) sound mode which exhibits a minimal shear viscosity $\eta = s/4\pi$.
- The absence of other ballistic modes is caused by a cascade of “democratic” splittings of the energy of any excited state. Think of squirting a *super soaker* water gun under water!
- Only the space-like sound mode is protected by conservation laws.

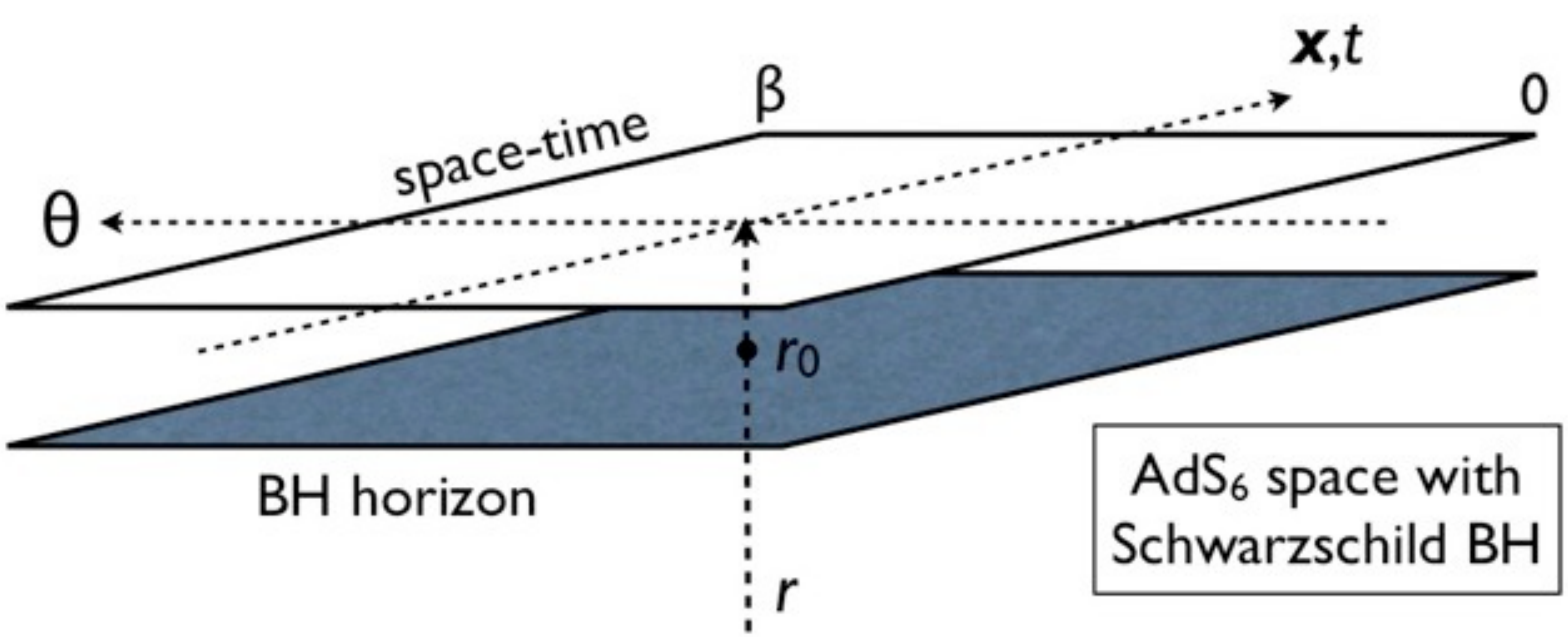
A holographic realization?

- The (3+1)-dimensional N = 4 SYM theory is the holographic dual of a string theory on the $AdS_5 \times S_5$ background.
- AdS_6 space with a black brane may give the possible realization of a holographic dual with 2 time dimensions, one of which is thermal.

$$ds^2 = \frac{r^2}{R^2}\left[f(r)d\theta^2 - dt^2 + d\mathbf{x}^2\right] + \frac{R^2}{f(r)r^2}\,dr^2$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^5$$

$$\beta_5 = \frac{4\pi R^2}{5r_0}$$



A simpler case study

Consider SU(2) gauge theory in (3+2) dimensions under the following conditions:

- the second time dimension θ is thermalized at temperature T_5;
- the physical time dimension t is at zero temperature.

We can consider the theory in imaginary physical time $t = -i\tau$, which corresponds to the gauge theory in (4+1) dimensions and then continue analytically to real t.

[An aside: This theory is not renormalizable, but it can be defined on a lattice. This may not be a serious concern for us (for reasons to be discussed).]

Field strength tensor:

$$
\begin{array}{c} \downarrow \rightarrow \\ x \\ y \\ z \\ \tau \\ \theta \end{array}
\begin{array}{c} \begin{array}{ccccc} x & y & z & \tau & \theta \end{array} \\
\begin{pmatrix}
0 & B_z & -B_y & E_x & P_x \\
-B_z & 0 & B_x & E_y & P_y \\
B_y & -B_x & 0 & E_z & P_z \\
-E_x & -E_y & -E_z & 0 & P_\tau \\
-P_x & -P_y & -P_z & -P_\tau & 0
\end{pmatrix}
\end{array}
$$

5-dim. "magnetic" sector

5-dim. "electric" sector

Dimensional reduction

This theory describes a (4+1)-dimensional “gluon plasma” characterized by a coupling constant g_5 with dimension $(\text{mass})^{-1/2}$.

$$
\begin{array}{c} \downarrow \\ x \\ y \\ z \\ \tau \\ \theta \end{array}
\begin{array}{c} \rightarrow \quad x \qquad y \qquad z \qquad \tau \qquad \theta \\
\begin{pmatrix}
0 & B_z & -B_y & E_x & P_x \\
-B_z & 0 & B_x & E_y & P_y \\
B_y & -B_x & 0 & E_z & P_z \\
-E_x & -E_y & -E_z & 0 & P_\tau \\
-P_x & -P_y & -P_z & -P_\tau & 0
\end{pmatrix}
\end{array}
$$

5-dim. “magnetic” sector ⇒ θ-time dependent fields screened; static magnetic fields remain unscreened

5-dim. “electric” sector ⇒ All electric fields are Debye screened

Screening length in (x,y,z,τ) space is given by $(g_5 T_5^{3/2})^{-1}$. If this is very small ($< 10^{-18}$ cm) then physics on presently accessible length scales is completely described by fields ***E***, ***B***, which are independent of θ. Furthermore, their dynamics is that of the SU(2) gauge theory in 4 euclidean dimensions (x,y,z,τ) with coupling constant $g_4 = g_5 T_5^{1/2}$.

We can now analytically continue τ to real time t and obtain the non-abelian gauge theory in the vacuum of (3+1)-dimensional Minkowski space.

Classical to quantum (I)

While this is interesting enough - a thermalized second time dimension could be “hidden” from view - it suggests an even more exciting possibility.

Assume that the 5-dimensional gauge field is not a quantum field but a *classical* field, and that it is in a state of high energy density w.r.t. the θ-energy component. The self-interacting gauge field obeys a nonlinear and highly chaotic dynamics, which ergodically covers the space of field configurations of a given energy E_θ.

The field quickly looks *equilibrated* in the micro-canonical sense. For a system with very many degrees of freedom, such as the gauge field, the micro-canonical ensemble is indistinguishable from the canonical ensemble, and the energy density E_θ/V is equivalent to a temperature T_5.

The expectation value of any quantity sensitive only to long distances in (x,y,z,τ) space can then be expressed as a thermal expectation value

$$\langle \mathcal{O} \rangle = \int [dA]\, \delta(E_\theta[A] - E_\theta)\, \mathcal{O}[A] \sim \int [dA]\, e^{-E_\theta[A]/T_5}\, \mathcal{O}[A]$$

Classical to quantum (II)

Now E_θ is just the 4-dimensional action S_4 of the gauge field in euclidean (x,y,z,τ) space, and thus we can write

$$\langle \mathcal{O} \rangle \sim \int [dA]\, e^{-E_\theta[A]/T_5}\, \mathcal{O}[A] = \int [dA^{(4)}]\, e^{-S_4[A]/(T_5 a)}\, \mathcal{O}[A^{(4)}]$$

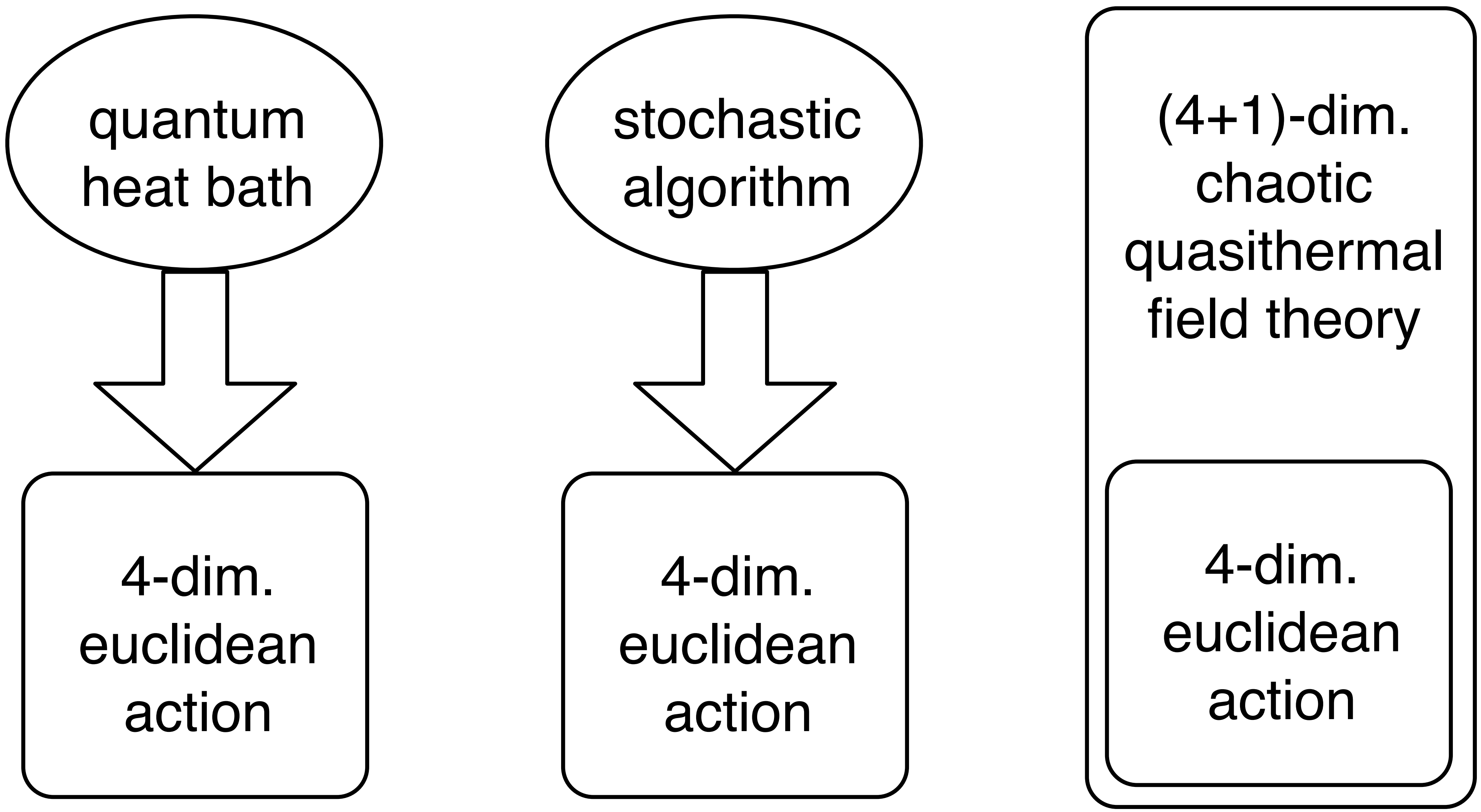
where a is a short-distance cut-off, e.g. a lattice spacing. But this is precisely the euclidean function integral for the 4-dimensional gauge theory, if we identify

$$\boxed{T_5 a \quad \leftrightarrow \quad \hbar}$$

In other words, the quasi-thermal dynamics of the non-abelian gauge field in the additional time dimension generates the quantum dynamics of the four-dimensional gauge field in its vacuum state.

This is a dynamical realization of stochastic quantization (Parisi & Wu 1981) or micro-canonical quantization (Strominger 1983). Corrections to the exact quantum dynamics are, in principle, calculable.

Quantization schemes



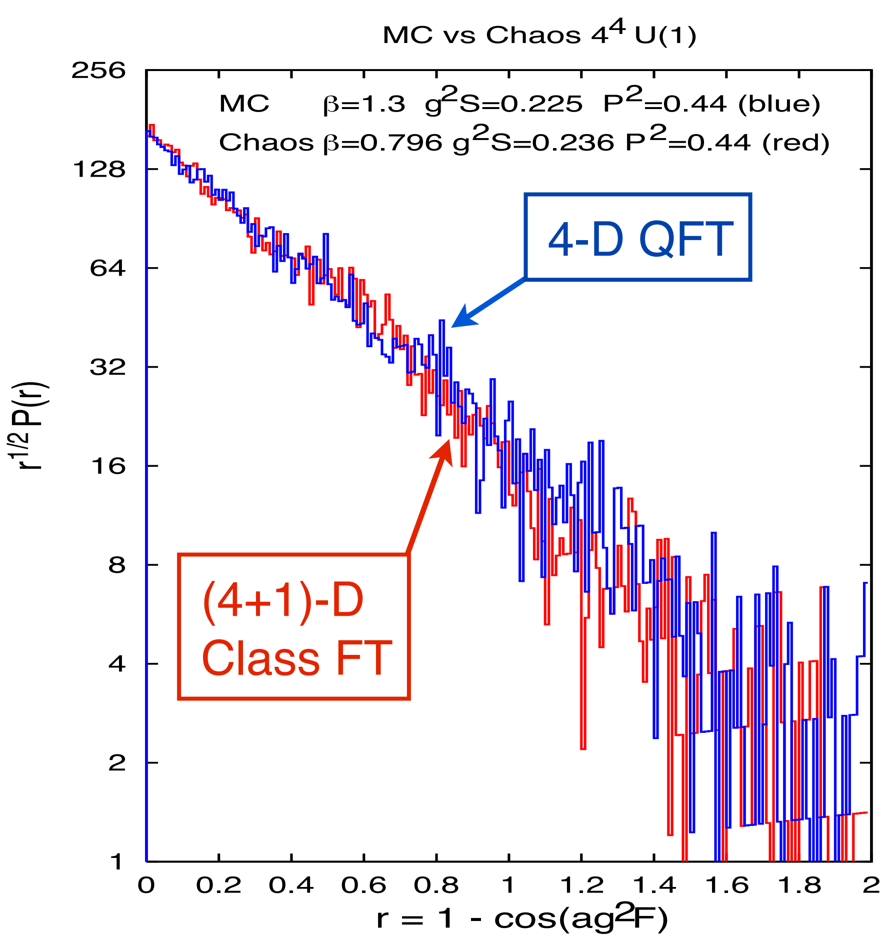
Canonical Quantization

Stochastic Quantization

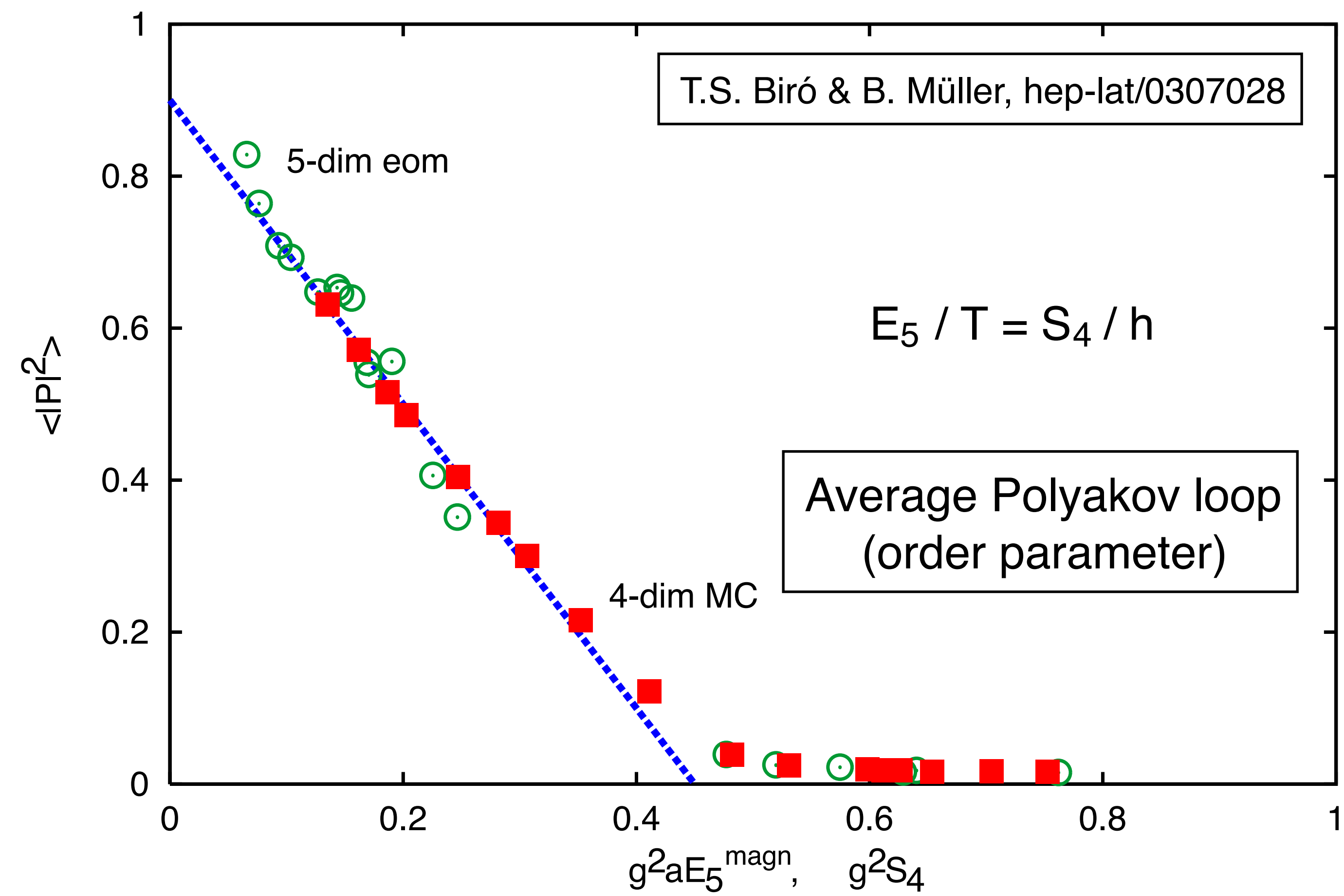
Dynamical (chaotic) Quantization

A reality check

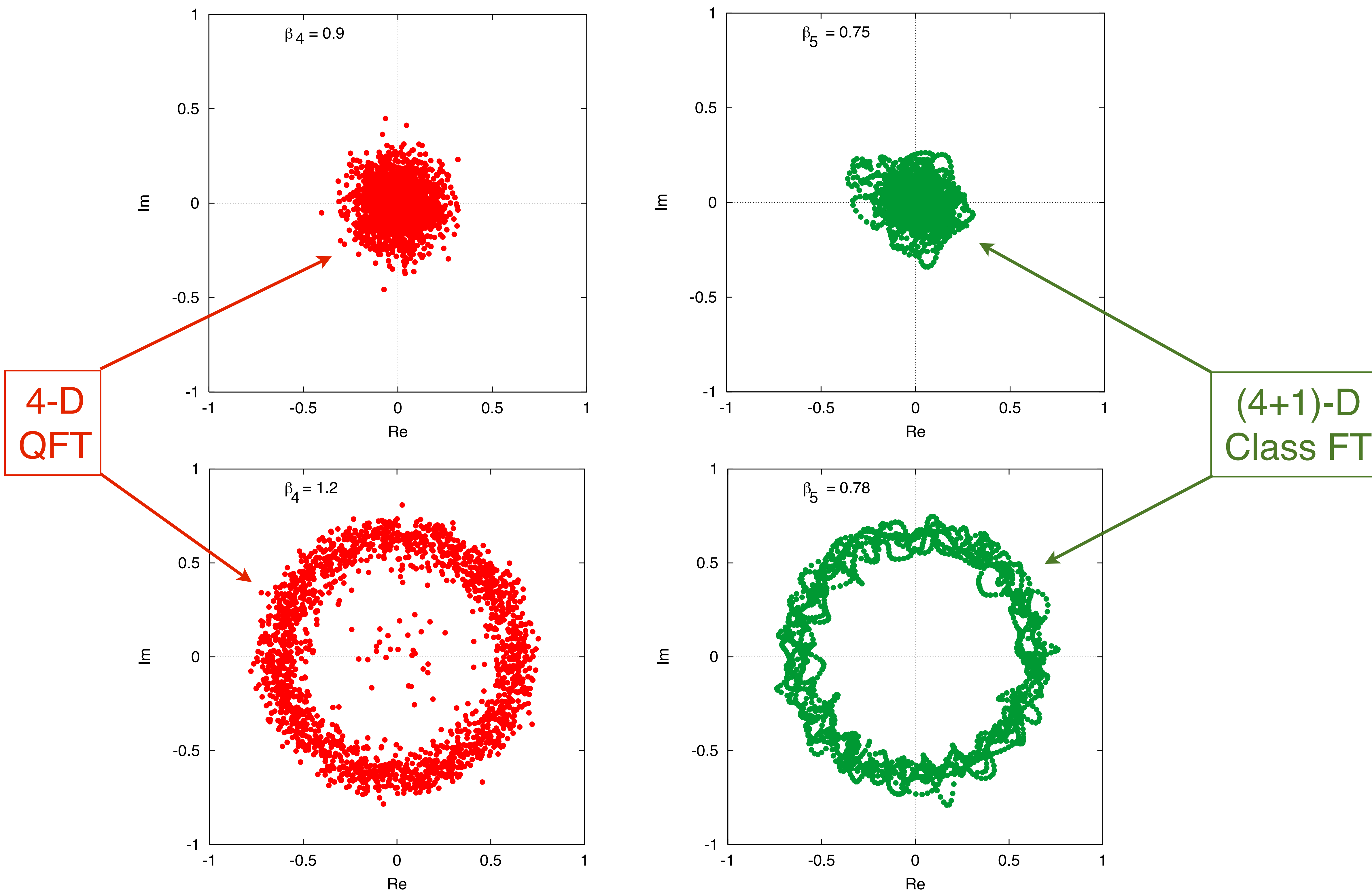
- Compact U(1) gauge theory on a lattice
  - Classical (4+1)-dimensional gauge theory with ergodic sampling
  - 4-dimensional quantum gauge theory with canonical MC sampling
- Correspondence: $g^2 a \langle E_5^{mag} \rangle \Leftrightarrow g^2 \langle S_4 \rangle$ or $(T_5\, a) \Leftrightarrow \hbar$



Plaquette energy distribution



QFT - CFT correspondence: $\langle L_P \rangle$



Taking stock

- A second time dimension seems to be able to avoid the problem of macroscopic causality violation due to the existence of time-like loops if the dynamics in it is sufficiently chaotic and/or thermal.
- The solution of the initial value problem in the physical time requires the absence of freely propagating excitations in the second time dimension.
- For non-abelian gauge fields, the long-distance dynamics of the thermal (4+1)-dimensional theory corresponds to the 4-dimensional gauge theory in the vacuum state.
- ***Conjecture***: If the (3+2)-dimensional *classical* field theory is sufficiently chaotic to produce a quasi-thermal ensemble on long second-time scales, it can generate the (3+1)-dimensional vacuum *quantum* dynamics with the correspondence $\hbar = T_5 a$, where a is a short-distance cut-off.

Are all problems resolved ?

1. Unitarity of the scattering matrix (Ynduráin 1991):

For a spin-0 particle scattering on a static point source in the presence of a second, time dimension of period *L*, the scattering amplitude in 1st order Born approximation

$$T_{\rm Born}(\mathbf{k}) = -\frac{g^2}{4\pi^2}\sum_{n\in Z}\frac{1}{\mathbf{k}^2 - n^2/L^2 - i\varepsilon}$$

corresponds to a complex potential

$$V(r) = \int \frac{d\mathbf{k}}{2\pi} e^{i\mathbf{k}\cdot\mathbf{r}}\, T_{\rm Born}(\mathbf{k}) = -\frac{g^2}{4\pi r}\left(1 + 2\sum_{n=1}^{\infty} e^{-inr/L}\right)$$

If the second time dimension, instead of being periodic, is thermal at temperature T_5 , the potential is real with exponentially suppressed corrections:

$$V(r) = -\frac{g^2}{4\pi r}\left(1 + 2\sum_{n=1}^{\infty} e^{-nT_5 r}\right)$$

just as for an additional curled-up space-like dimension.

Are all problems resolved ?

2. Existence of tachyonic modes:

These arise from the 5-dimensional dispersion relation $(\omega_1)^2 = \mathbf{k}^2 + m^2 - (\omega_2)^2$ which yields exponentially growing modes for large ω_2 .

In the quantum field theory, we can argue that for long times the ergodic average equals the thermal average, and that is obtained by summing over Matsubara frequencies

$$\omega_2^{(n)} = i2n\pi T_5 \qquad \Longrightarrow \qquad \left(\omega_1^{(n)}\right)^2 = \mathbf{k}^2 + m^2 + (2n\pi T_5)^2$$

Alternatively, if the (3+2)-dimensional field theory is classical and nonlinear, exponentially growing modes are not problematic (they correspond to positive Lyapunov exponents) as long as the theory is chaotic and all correlations functions fall off exponentially with (second) time.

Are all problems resolved ?

3. Existence of ghost modes:

This is a problem for vector fields, because in (3+2)-dimensions the Feynman propagator

$$D_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon}$$

has 2 modes with the wrong sign in the numerator (for g_{tt} and $g_{\theta\theta}$). One of these is eliminated by charge conservation / Gauss' law, but the second one due to $g_{\theta\theta}$ is a problem.

In a recent study of the Lee-Wick $O(N)$ model, which contains a scalar ghost field, Grinstein, O'Connell and Wise (PRD79, 105019) showed that causality emerges intact on macroscopic time scales, when the ghost field corresponds to a rapidly decaying resonance. Since all "electric" field excitations (the $g_{\theta\theta}$-mode) decay rapidly in the thermal (3+2)-dimensional gauge theory, the same principle may apply (but this is an unproven conjecture!).

Open questions

- Is there a fully consistent QFT in (3+2) dimensions, or does only a classical field theory with UV cut-off exist in a strict sense?
- Is it possible to construct a (3+2)-dimensional field theory, which reduces to a (3+1)-dimensional theory with fermions when the second time dimension is thermal?
- Is it possible to construct a consistent (3+2)-dimensional field theory which reduces to the standard model in the low-energy limit?
- What are the precise limits on T_5 from experimental data? Which observables are most sensitive?
- How does the temperature T_5 in the second time dimension arise? Is it of cosmological origin, e.g. in an ekpyrotic big-bang scenario?
- Why does T_5 not decrease as the universe expands?
- What does it mean to have a very large T_5 and a small $T_4 = 2.73\ K$ at the same time?

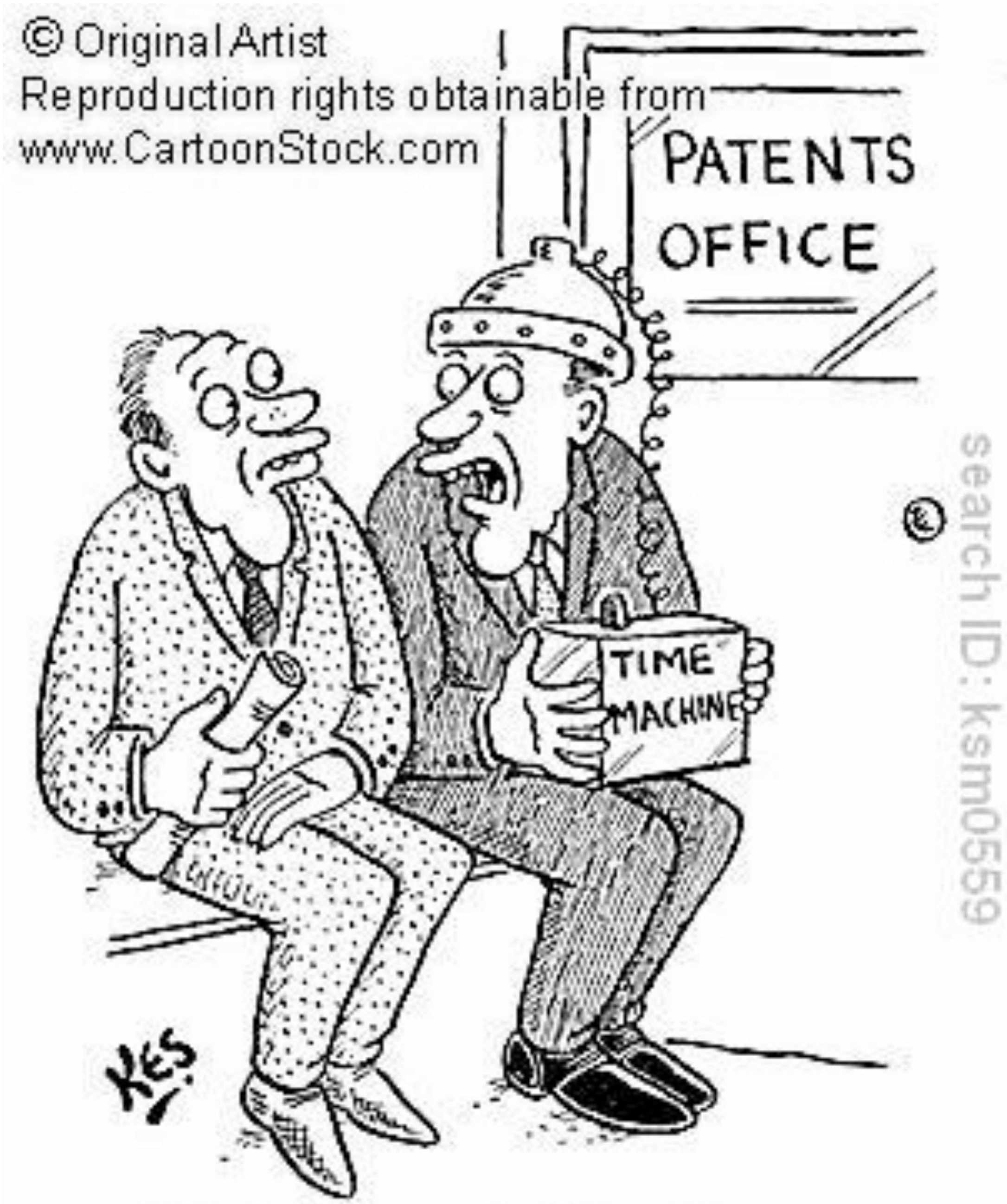
A final thought

Investigating the viability and potential physical usefulness of field theories in space-times with 2 time-like dimensions is fraught with many risks.

However, besides the intellectual excitement, there may be two potential benefits:

- An explanation of how quantum physics arises from classical dynamics. [Maybe gravity must not be quantized, after all?]
- A natural explanation for why supersymmetry is not realized at low energies.

Many questions remain and much work needs to be done!



"Don't get too excited. Your idea never really gets off the ground."

The END