Problem 1 [4 pts]
Text problem 4.6 (see handout on electron degeneracy pressure for the equation required in addition to the discussion on white dwarfs in the text (page 123)).

Problem 2 [4 pts]
As discussed in class, the *Schwarzschild radius* defines the *Event Horizon* of an object if all the mass of the object were located at a single point (a singularity) in space. It is the radius inside which nothing can be transmitted to the universe outside. Calculate the Schwarzschild radius for **(a)** a black hole of mass 40 times that of our Sun, **(b)** the Sun, **(c)** the Earth, and, **(d)** a proton.

Problem 3 [6 pts]
Consider a white dwarf of mass 2 solar masses, and radius $R = 3 \times 10^8$ m. Using the Schwarzschild metric calculate at the surface of the white dwarf: **(a)** The gravitational refshift, **(b)** the time dilation, and, **(c)** the radius of curvature of the path of an initially horizontally travelling photon. (This last part is a bit tricky, but it will show you how much a photon's path deviates from a straight line (which has infinite radius of curvature)).

Problem 4 [4 pts]
Text problem 5.2 (for properties of the Sun see for example: `http://en.wikipedia.org/wiki/Sun`)

Problem 5 [4 pts]
Text problem 5.4

Problem 6 [2 pts]
How do the values obtained in problems 4 and 5 compare to those obtained using a White Dwarf as the starting point for a Neutron Star (see class notes)? Why wouldn't they necessarily agree ?

Problem 7 [20 pts]

In class we obtained the following differential equation that we need to solve to get the scale factor as a function of time, $R(t)$:

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right] R^2 = -kc^2$$

where R and ρ are both functions of t.

(a) Show that we can rewrite this in terms of R and t only thus:

$$\left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho_0\frac{1}{R} = -kc^2$$

where ρ_0 is a constant, the average density of the present universe.

(b) From this equation, verify the following solutions for $R(t)$ for a flat, closed, and open universe:

$$R(t) = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3} \qquad \text{(for k = 0)}$$

$$R(t) = \frac{1}{2}\frac{\Omega_0}{\Omega_0 - 1}(1 - \cos x) \quad \text{where,} \quad t = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}(x - \sin x) \qquad \text{(for k > 0)}$$

$$R(t) = \frac{1}{2}\frac{\Omega_0}{1 - \Omega_0}(\cosh x - 1) \quad \text{where,} \quad t = \frac{1}{2H_0}\frac{\Omega_0}{(1 - \Omega_0)^{3/2}}(\sinh x - x) \qquad \text{(for k < 0)}$$

For the flat universe case explicitly solve the differential equation for R above, but for the closed and open universe cases it is sufficient to simply substitute these solutions into the differential equation to show they do in fact solve it.

(c) Sketch the form of these solutions on a plot of $R(t)$ versus t.

(d) Show that from these solutions we can obtain the time(age) of the universe as function of the redshift z thus (this is a bit messy, sorry!):

$$\frac{t(z)}{t_H} = \frac{2}{3}\frac{1}{(1+z)^{3/2}} \qquad \text{(for k = 0)}$$

$$\frac{t(z)}{t_H} = \frac{\Omega_0}{2(\Omega_0 - 1)^{3/2}}\left[\cos^{-1}\left(\frac{\Omega_0 z - \Omega_0 + 2}{\Omega_0 z + \Omega_0}\right) - \frac{2\sqrt{(\Omega_0 - 1)(\Omega_0 z + 1)}}{\Omega_0(1+z)}\right] \qquad \text{(for k > 0)}$$

$$\frac{t(z)}{t_H} = \frac{\Omega_0}{2(1 - \Omega_0)^{3/2}}\left[-\cosh^{-1}\left(\frac{\Omega_0 z - \Omega_0 + 2}{\Omega_0 z + \Omega_0}\right) + \frac{2\sqrt{(1 - \Omega_0)(\Omega_0 z + 1)}}{\Omega_0(1+z)}\right] \qquad \text{(for k < 0)}$$

where Ω_0 is the density parameter of the present universe, as defined in class.

(e) For density parameters of $\Omega_0 = 1$, $\Omega_0 = 2$, and $\Omega_0 = 0.5$, calculate the age of the universe for each as ratio of t_H ($t_H = 13.6$ billion years for $h = 0.72$).

Problem 8 [6 pts]
(a) For the closed universe scenario derive an expression for the maximum "radius" (R_{max}) of the universe.

(b) If the density of the universe was twice the critical density, ρ_c, by what factor would the universe expand beyond its present radius, and what would be the time between the "Big Bang" and the "Big Crunch" ?

Problem 9 [4 pts]
Assuming that our knowledge of particle physics is uncertain at energies greater than about $10^3 GeV$, find the earliest time t at which we can have confidence in the physics that existed in the early universe. How much smaller was the universe at that time than it is now ?

Problem 10 [6 pts]
Assuming Helium nuclei were being formed when the temperature of the Universe was about $10^9 K$, estimate the time(t), scale factor ($R(t)$), and redshift (z), of the Universe when this was occuring.