

Problem 1 [6 pts]

Consider a star late in its evolution formed with an initial composition of $X_H = 0.70$, $X_{He} = 0.28$, and $X_{other} = 0.02$, where “other” refers to elements that are much heavier than H or He. Assuming no contribution to the pressure gradient from electron degeneracy, estimate (using the Schönberg-Chandrasekhar limit) the ratio of the mass of the inner Helium core to that of the whole star just before the inner core starts to collapse.

(Hints: In calculating μ_{ic} assume that the inner core is entirely helium, and that the temperature is high enough (which it easily will be) such that complete ionization can be assumed – note that this means when you calculate μ you must consider the total number of particles as being from both nuclei and electrons. Also, assume complete ionization in the H burning layer in calculating μ_e , but using the mass fractions given. Also recall that the mean molecular weight is defined by $\mu \equiv \bar{m}/m_H$).

Problem 2 [12 pts]

(a) Calculate the total gravitational potential energy of a star of mass M , and radius R . An accurate calculation would require knowing ρ as a function of r , but here assume that ρ is constant and equal to its average value of $\bar{\rho} = M/\frac{4}{3}\pi R^3$. This will be easiest to calculate if you write down the potential energy of a shell of mass dm at radius r due to the gravitational force between dm and M_r , the mass inside radius r , and then integrate this expression from $r = 0$ to $r = R$.

(b) What is the total mechanical energy, E , of the star ?

(c) Let this star be the Sun, with $M = M_\odot$ and $R = R_\odot$. Assume that the Sun was formed from a large cloud of hydrogen gas with an initial radius $R \gg R_\odot$. Assume that the loss in total mechanical energy during this collapse is radiated away. What then is the energy radiated away in joules from the gravitational collapse ?

(d) Assuming that the Sun’s luminosity, $L_\odot \approx 4 \times 10^{40} \text{Js}^{-1}$, has been roughly constant throughout its lifetime, how long could it radiate at this rate given the available energy calculated in (c) ? This is often called the Kelvin-Helmholtz time-scale, t_{KH} , and gives a rough scale for stellar formation from gas clouds.

Problem 3 [6 pts]

Describe in your own words *electron degeneracy pressure* and its importance in stellar interiors for stars at the end of their main-sequence lives. You are encouraged to conduct your own research into the phenomenon in addition to using my notes. If you use other sources make sure to reference them. Keep your discussion to about a page.

Problem 4 [6 pts]

Text problem 4.5