Problem 1 [9 pts]
Consider the collision of a photon with wavelength λ_i with an electron of mass m_e at rest. After the collision the scattered photon makes an angle θ with respect to the initial photon direction, and has wavelength λ_f. Write down the conservation of relativistic energy and momentum equations and using these derive the Compton scattering equation:

$$\Delta\lambda \;=\; \lambda_f - \lambda_i \;=\; \frac{h}{m_e c}(1-\cos\theta)$$

Problem 2 [3 pts]
Calculate the energies (in eV) and wavelengths (in nm) of all possible photons that are emitted when an electron transitions from the $n = 3$ to the $n = 1$ orbit of the hydrogen atom ? (Consider the 3 different photons in total from the two different possible paths from the $n = 3$ to the $n = 1$ orbit.)

Problem 3 [3 pts]
Show that, at room temperature, the thermal energy $kT \approx 1/40\,\mathrm{eV}$. At what temperature is kT equal to 1 eV ? to 13.6 eV ?

Problem 4 [2 pts]
The cosmos is pervaded by a 3 K radiation field, which is regarded as the "echo" of the Big Bang. This radiation field is called the Cosmic Microwave Background. Calculate the energy and wavelength of this radiation.

Problem 5 [3 pts]
In its rest frame, quasar Q2203+29 produces a hydrogen emission line of wavelength 121.6 nm. Astronomers on Earth measure a wavelength of 656.8 nm for this line. Determine the redshift parameter (z) and the speed of recession for this quasar.

Problem 6 [3 pts]
Quasar 3C 446 is extremely variable: its luminosity at optical wavelengths has been observed to change by a factor of 40 in as little as 10 days. Using the redshift parameter $z = 1.404$ measured for 3C 446, determine the time for the luminosity variation as measured in the quasar's rest frame.

Problem 7 [7 pts]
Consider a gas of neutral hydrogen atoms.

(a) At what temperature will equal numbers of atoms have electrons in the ground state and in the second excited state ($n = 3$) ?

(b) At a temperature of 85,400 K, when an equal number (N) of atoms are in the ground state and in the first excited state, how many atoms are in the second state ($n = 3$) ? Express your answer in terms of N.

(c) As the temperature $T \to \infty$, how will the electrons in the hydrogen atom be distributed, according to the Boltzmann equation ? That is, what will be the relative numbers of electrons in the $n = 1, 2, 3, ...$ orbitals ?