

Problem 1 [2 pts]

The lifetime of the particle called the pi meson (or pion), is $\tau_\pi = 2.5 \times 10^{-8}$ s when the pion is at rest relative to the observer measuring its decay time. What is the lifetime measured by an observer at rest for pions travelling with a speed of $v = 0.999c$?

Problem 2 [4 pts]

Text problem 1.4

Problem 3 [4 pts]

Text problem 1.6

Problem 4 [6 pts]

(a) Write the Lorentz transformations relating (t', x', y', z') and (t, x, y, z) .

(b) Show by explicit computation that the proper time interval, $d\tau$, where;

$$c^2 d\tau^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)$$

is invariant under Lorentz transformations. That is, $d\tau^2 = (d\tau')^2$.

(c) Introduce the notation $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = v/c \equiv \tanh u$, and also define $x_0 = ict$. Rewrite the Lorentz transformations in part (a) in terms of $\cosh u$ and $\sinh u$.

(d) The results in part (c) can be recast as a matrix equation, $x'^i = \Lambda_j^i x^j$, where $x^i = (x_0, x, y, z)$. Write Λ_j^i as an explicit 4×4 matrix.

(**Comment:** By using the results of part (d) and the relations $\sin iu = i\sinh u$, $\cos iu = i\cosh u$, it can be seen that the Lorentz transformations can be viewed as a “rotation” in the $x_0 - x$ plane through an (imaginary) angle iu . In this representation, the time coordinate x_0 joins the space coordinates on an equal footing.)

Problem 5 [4 pts]

In class we derived the relativistic form of the kinetic energy to be:

$$K = mc^2(\gamma - 1)$$

Show that this reduces to the familiar $K = \frac{1}{2}mv^2$ for $v \ll c$ (the Newtonian limit).

Problem 6 [4 pts]

Text problem 1.9

Problem 7 [6 pts]

Text problem 1.10