

The first six questions involve concepts and techniques that you should be familiar with, or that you will need to review if necessary.

**Problem 1 [5 pts]**

(a) For a particle of mass  $m$  moving with constant speed  $v$  in a circular orbit of radius  $r$  from the center, derive the expression for the centripetal acceleration  $a_c$  in terms of  $v$  and  $r$ .

(b) Assume the Moon's orbit is exactly circular, its distance from the centre of the Earth is  $r = 3.844 \times 10^8$  m, and its orbital period is 27.3 days. Calculate the speed  $v$  of the moon, and its centripetal acceleration  $a_c$ .

(c) Calculate the acceleration of the Moon caused by the Earth's gravitational pull. The mass of the Earth is  $M_{\oplus} = 5.974 \times 10^{24}$  kg. How does this compare to the centripetal acceleration calculated in (b) ?

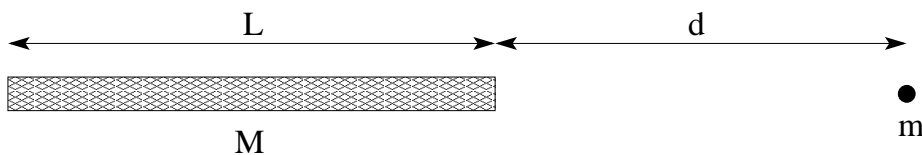
**Problem 2 [3 pts]**

Given that the Sun takes about  $2 \times 10^8$  years to make a circular orbit around the Galactic Centre, at a distance of 10 kpc, estimate the mass of the Galaxy contained within the solar orbit, assuming that it is spherical and ignoring any effects of mass lying outside the orbit.

(Note: the unit "kpc" is a *kilo-parsec*, and is derived in Problem 7. See also the conversions attached to this problem set.)

**Problem 3 [4 pts]**

Calculate the Gravitational potential energy between the point mass  $m$  shown in the figure, and a uniform thin rod of mass  $M$  (in terms of  $G$ ,  $m$ ,  $M$ ,  $d$ , and  $L$ ).



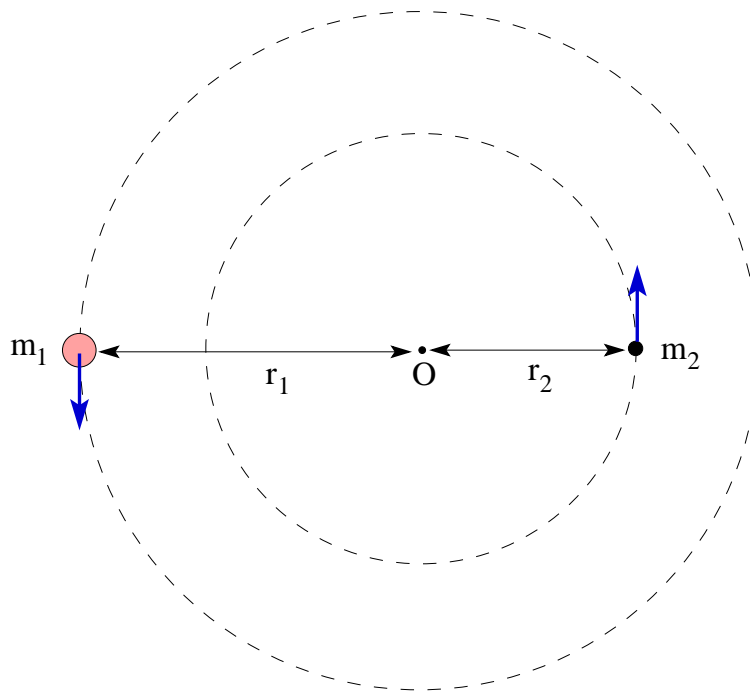
**Problem 4 [5 pts]**

Review and discuss Kepler's three laws of Planetary motion. Keep it to about a page in total.

**Problem 5 [5 pts]**

One particular binary star system consists of a visible star of mass  $m_1 = 5M_\odot$  where  $M_\odot$  is the mass of the Sun, and an unseen black hole whose mass,  $m_2$ , we would like to determine. The visible star has an orbital speed of  $v_1 = 300 \text{ km/s}$  and an orbital period of  $T = 1.2 \text{ days}$ . Assuming both stars have circular orbits as shown, determine  $m_2$  (express both in kg and  $M_\odot$ ).

[Hint: Note that the two stars will not be in circular orbits about each other, but rather about the centre of mass of the two-star system, indicated by “O” in the figure. Start by calculating  $r_1$  and  $r_2$ , the orbital radii about the centre of mass of the two stars, and then write down Newton’s second law for the star  $m_1$  noting that it is moving in a circular orbit of radius  $r_1$  caused by the gravitational force of  $m_2$  a distance  $r_1 + r_2$  from it.]

**Problem 6 [2 pts]**

What is the total angular momentum about the centre of mass of the two-star system discussed in problem 5 ?

**Problem 7 [3 pts]**

Take the diameter AB of the Earth's orbit as  $3 \times 10^8$  km and consider a star S at a distance  $d$ , such that SA = SB and the angle ASB = 2 arcseconds. Calculate  $d$ . This is the distance unit of one *parsec*. Relate it to one light year.

**Problem 8 [3 pts]**

To get a feel for distance scales in the universe, what are the following distances in parsecs (pc). (You'll need to look up some of these distances yourselves, and some are given in the attachment of constants and conversion factors.)

- (a) Diameter of a proton (in SI units it is about  $1 \text{ fm} = 10^{-15} \text{ m}$  (femtometer))
- (b) Radius of the Earth
- (c) Distance between the Earth and Sun
- (d) Size (diameter) of the Milky Way
- (e) Distance from the Milky Way to the next nearest galaxy
- (f) Distance from the Milky Way to the furthest known galaxy  
(thought to be "Abell 1835 IR1916" – google this to get more information)

### Useful constants and conversions

Speed of light	$c = 2.99792458 \times 10^8 \text{ m/s}$
Planck's constant	$h = 6.626 \times 10^{-34} \text{ Js}$
	$\hbar = \frac{h}{2\pi}$
Gravitational constant	$G = 6.672 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J/K}$
Radius of Sun	$R_{\odot} = 6.96 \times 10^8 \text{ m}$
Mass of Sun	$M_{\odot} = 1.989 \times 10^{30} \text{ kg}$
Radius of Earth	$R_{\oplus} = 6.38 \times 10^6 \text{ m}$
Mass of Earth	$M_{\oplus} = 5.974 \times 10^{24} \text{ kg}$
Average Earth to Moon distance	$r_{EM} = 3.84 \times 10^8 \text{ m}$
Average Earth to Sun distance	$r_{ES} = 1.496 \times 10^{11} \text{ m} = 1 \text{ AU (Astronomical Unit)}$
Mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511\text{MeV}/c^2$
Mass of proton	$m_p = 1.67 \times 10^{-27} \text{ kg} = 938.279\text{MeV}/c^2$
Mass of neutron	$m_n = 1.68 \times 10^{-27} \text{ kg} = 939.573\text{MeV}/c^2$

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$$1 \text{ J} = 10^7 \text{ erg} = 0.239 \text{ cal}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$1 \text{ keV} = 10^3 \text{ eV}, 1 \text{ MeV} = 10^6 \text{ eV}, 1 \text{ GeV} = 10^9 \text{ eV}$$

$$1 \text{ light year (lyr)} = 9.4605 \times 10^{15} \text{ m}$$

$$1 \text{ Parsec (pc)} = 3.0856 \times 10^{16} \text{ m} = 3.26 \text{ lyr}$$