Problem 1

(a) For the closed universe scenario derive an expression for the maximum "radius" (R_{max}) of the universe.

(b) If the density of the universe was twice the critical density, ρ_c, by what factor would the universe expand beyond its present radius, and what would be the time between the "Big Bang" and the "Big Crunch" ?

Problem 2

Assuming that our knowledge of particle physics is uncertain at energies greater than about $10^3 GeV$, find the earliest time t at which we can have confidence in the physics that existed in the early universe. How much smaller was the universe at that time than it is now ?

Problem 3

Assuming Helium nuclei were being formed when the temperature of the Universe was about $10^9 K$, estimate the time(t), scale factor ($R(t)$), and redshift (z), of the Universe when this was occuring.

Problem 4

As we have discussed, as the universe expands larger regions of the universe come into causal contact with a given observer. The farthest observable point in the universe is called the **particle horizon**, which, at a time t, has a proper distance $d_P(t)$. (Assuming the Friedmann models of the Universe, two points separated by more than d_P are not in causal contact.)

(a) In a Robertson-Walker metric, with a curvature k, what is the proper distance, $d(t)$, of an object from Earth at time t ? (Leave in terms of an integral over r, the coordinate distance of the object.). $d_P(t)$ is then the upper limit of $d(t)$ obtained by setting the lower limit of the integral over r (or t) at zero.

(b) During a radiation dominated era, assuming a flat universe, derive an expression for $d_P(t)$.

(c) During a matter dominated era, assuming a flat universe, derive an expression for $d_P(t)$. Rewrite this expression as $d_P(z)$, in terms of the redshift, z.
(Assume the matter dominated era comprises most of the time since the Big Bang so that one can still set the lower limit to $t = 0$ for these estimates.)

(d) Estimate the distance (give the value) to the particle horizon in a flat universe at the present time.

Problem 5

Assuming that the present density of ordinary baryonic matter is $\rho_{B,0} = 3 \times 10^{-28} kg\, m^{-3}$, what was the density of matter at the time of *Big Bang nucleosynthesis*, when $T \sim 10^{10} K$?