

Figure 20.8 The circuit in this flashlight consists of a resistor (the filament of the light bulb) connected to a 3.0-V battery.

20.3 RESISTANCE AND RESISTIVITY

In a water pipe, the length and cross-sectional area of the pipe determine the resistance that the pipe offers to the flow of water. Longer pipes with smaller cross-sectional areas offer greater resistance. Analogous effects are found in the electrical case. For a wide range of materials, the resistance of a piece of material of length L and cross-sectional area A is

$$R = \rho \frac{L}{A} \quad (20.3)$$

where ρ is a proportionality constant known as the *resistivity* of the material. It can be seen from Equation 20.3 that the unit for resistivity is the ohm·meter ($\Omega \cdot \text{m}$), and Table 20.1 lists values for various materials. All the conductors in the table are metals and have small resistivities. Insulators such as rubber have large resistivities. Materials like germanium and silicon have intermediate resistivity values and are, accordingly, called *semiconductors*.

Resistivity is an inherent property of a material, inherent in the same sense that density is an inherent property. Resistance, on the other hand, depends on both the

Table 20.1 Resistivities^a of Various Materials

Material	Resistivity ρ ($\Omega \cdot \text{m}$)	Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Conductors		Semiconductors	
Aluminum	2.82×10^{-8}	Carbon	3.5×10^{-5}
Copper	1.72×10^{-8}	Germanium	0.5^b
Gold	2.44×10^{-8}	Silicon	20–2300 ^b
Iron	9.7×10^{-8}	Insulators	
Mercury	95.8×10^{-8}	Mica	$10^{11} - 10^{15}$
Nichrome (alloy)	100×10^{-8}	Rubber (hard)	$10^{13} - 10^{16}$
Silver	1.59×10^{-8}	Teflon	10^{16}
Tungsten	5.6×10^{-8}	Wood (maple)	3×10^{10}

^aThe values pertain to temperatures near 20 °C.

^bDepending on purity.

The Physics of...
Electrical extension cords.

resistivity and the geometry of the material. Thus, two wires can be made from copper, which has a resistivity of $1.72 \times 10^{-8} \Omega \cdot \text{m}$, but Equation 20.3 indicates that a short wire with a large cross-sectional area has a smaller resistance than does a long, thin wire. Wires that carry large currents, such as main power cables, are thick rather than thin so that the resistance of the wires is kept as small as possible. Similarly, electric tools that are to be used far away from wall sockets require thicker extension cords, as Example 3 illustrates.

EXAMPLE 3 • Longer Extension Cords

The instructions for an electric lawn mower suggest that a 20-gauge extension cord can be used for distances up to 35 m, but a thicker 16-gauge cord should be used for longer distances, to keep the resistance of the wire as small as possible. The cross-sectional area of 20-gauge wire is $5.2 \times 10^{-7} \text{ m}^2$, while that of 16-gauge wire is $13 \times 10^{-7} \text{ m}^2$. Determine the resistance of (a) 35 m of 20-gauge copper wire and (b) 75 m of 16-gauge copper wire.

Reasoning and Solution We can use Equation 20.3, along with the resistivity of copper from Table 20.1, to find the resistance of the wires:

$$\text{20-gauge wire } R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(35 \text{ m})}{5.2 \times 10^{-7} \text{ m}^2} = \boxed{1.2 \Omega}$$

$$\text{16-gauge wire } R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(75 \text{ m})}{13 \times 10^{-7} \text{ m}^2} = \boxed{0.99 \Omega}$$

Even though it is more than twice as long, the thicker 16-gauge wire has less resistance than the thinner 20-gauge wire. It is necessary to keep the resistance as low as possible in order to minimize heating of the wire, thereby reducing the possibility of a fire, as Conceptual Example 7 in Section 20.5 emphasizes.

The Physics of...
Impedance plethysmography.

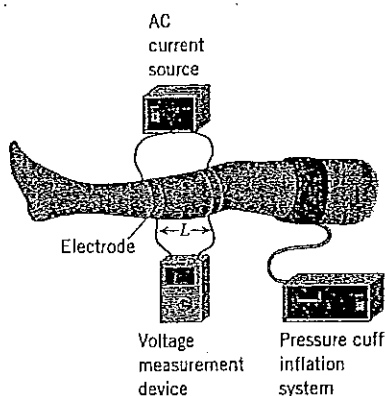


Figure 20.9 In the technique of impedance plethysmography, the electrical resistance of the calf is measured to diagnose deep venous thrombosis (blood clotting in the veins).

Equation 20.3 provides the basis for an important medical diagnostic technique known as impedance (or resistance) plethysmography. Figure 20.9 shows how the technique is applied to diagnose blood clotting in the veins (deep venous thrombosis) near the knee. A pressure cuff, like that used in blood pressure measurements, is placed around the midthigh, while electrodes are attached around the calf. The two outer electrodes are connected to a source of a small amount of ac current. The two inner electrodes are separated by a distance L , and the voltage between them is measured. The voltage divided by the current gives the resistance. The key to this technique is the fact that resistance can be related to the volume between the inner electrodes. The volume is the product of the length L and the cross-sectional area A of the calf, or $\text{Volume} = LA$. Solving for A and substituting in Equation 20.3 shows that

$$R = \rho \frac{L}{A} = \rho \frac{L}{\text{Volume}/L} = \rho \frac{L^2}{\text{Volume}}$$

Thus, resistance is inversely proportional to volume, a fact that is exploited in diagnosing deep venous thrombosis. Blood flows from the heart into the calf through arteries in the leg and returns through the system of veins. The pressure cuff in Figure 20.9 is inflated to the point where it cuts off the venous flow, but not the arterial flow. As a result, more blood enters than leaves the calf, the volume of the calf increases, and the electrical resistance decreases. When the cuff pressure is removed suddenly, the volume returns to a normal value, and so does the electrical resistance. With healthy (unclotted) veins, there is a rapid return to normal values. A slow return, however, reveals the presence of clotting.

The resistivity of a material depends on temperature. In metals, the resistivity increases with increasing temperature, whereas in semiconductors the reverse is true. For many materials and limited temperature ranges, it is possible to express the temperature dependence of the resistivity as follows:

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad (20.4)$$

In this expression ρ and ρ_0 are the resistivities at temperatures T and T_0 , respectively. The term α has the unit of reciprocal temperature and is the *temperature coefficient of resistivity*. When the resistivity increases with increasing temperature, α is positive, as it is for metals. When the resistivity decreases with increasing temperature, α is negative, as it is for the semiconductors carbon, germanium, and silicon. Since resistance is given by $R = \rho L/A$, both sides of Equation 20.4 can be multiplied by L/A to show that resistance depends on temperature according to

$$R = R_0[1 + \alpha(T - T_0)] \quad (20.5)$$

The next example illustrates the role of the resistivity and its temperature coefficient in determining the electrical resistance of a piece of material.

EXAMPLE 4 • The Heating Element of an Electric Stove

The Physics of...
a heating element on an electric stove.

Figure 20.10a shows a cherry-red heating element on an electric stove. The element contains a wire (length = 1.1 m, cross-sectional area = $3.1 \times 10^{-6} \text{ m}^2$) through which electric charge flows. As Figure 20.10b shows, this wire is imbedded within an electrically insulating material that is contained within a metal casing. The wire becomes hot in response to the flowing charge and heats the casing. The material of the wire has a resistivity of $\rho_0 = 6.8 \times 10^{-5} \Omega \cdot \text{m}$ at $T_0 = 320^\circ \text{C}$ and a temperature coefficient of resistivity of $\alpha = 2.0 \times 10^{-3} (\text{C}^\circ)^{-1}$. Determine the resistance of the heater wire at an operating temperature of 420°C .

Reasoning Equation 20.3 ($R = \rho L/A$) can be used to find the resistance of the wire at 420°C , once the resistivity ρ is determined at this temperature. Since the resistivity at 320°C is given, Equation 20.4 can be employed to find the resistivity at 420°C .

Solution At the operating temperature of 420°C , the material of the wire has a resistivity of

$$\begin{aligned} \rho &= \rho_0[1 + \alpha(T - T_0)] \\ \rho &= (6.8 \times 10^{-5} \Omega \cdot \text{m})[1 + (2.0 \times 10^{-3} (\text{C}^\circ)^{-1})(420^\circ \text{C} - 320^\circ \text{C})] \\ &= 8.2 \times 10^{-5} \Omega \cdot \text{m} \end{aligned}$$

This value of the resistivity can be used along with the given length and cross-sectional area to find the resistance of the heater wire:

$$R = \frac{\rho L}{A} = \frac{(8.2 \times 10^{-5} \Omega \cdot \text{m})(1.1 \text{ m})}{3.1 \times 10^{-6} \text{ m}^2} = \boxed{29 \Omega} \quad (20.3)$$

There is an important class of materials whose resistivity suddenly goes to zero below a certain temperature T_c called the *critical temperature*, commonly a few degrees above absolute zero. Below this temperature, such materials are called *superconductors*. The name derives from the fact that with zero resistivity, these materials offer no resistance to electric current and are, therefore, perfect conductors. One of the remarkable properties of zero resistivity is that once a current is established in a superconducting ring, it continues indefinitely without the need of an emf. Currents have persisted in superconductors for many years without measurable decay.

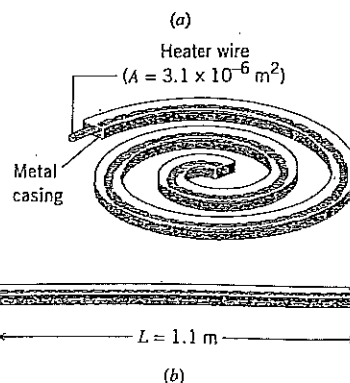
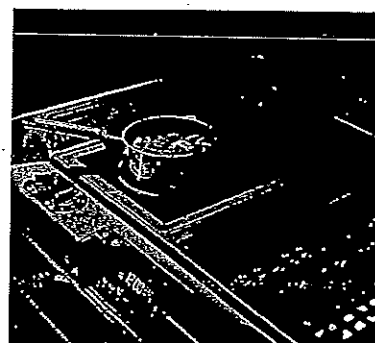


Figure 20.10 A heating element from an electric stove.

In contrast, the current in a nonsuperconducting material drops to zero almost immediately after the emf is removed.

Many metals become superconductors only at very low temperatures, such as aluminum ($T_c = 1.18$ K); tin ($T_c = 3.72$ K), lead ($T_c = 7.20$ K), and niobium ($T_c = 9.25$ K). Recently, ceramic materials have been developed that undergo the transition to the superconducting state at much higher temperatures. Transition temperatures as high as 133.5 K have been confirmed, while some evidence suggests that values as high as 250 K may be possible. The ultimate hope is to discover materials that are superconducting at room temperature. Superconductors have many technological applications, including magnetic resonance imaging (Section 21.7), magnetic levitation of trains (Section 21.9), cheaper transmission of electrical power, powerful (yet small) electric motors, and faster computer chips.

20.4 ELECTRIC POWER

Suppose that an amount of charge Δq emerges from a battery in a time Δt and that the potential difference between the battery terminals is V . According to the definition of electric potential given in Equation 19.3, the energy of this charge is $(\Delta q)V$. Since the amount of energy per unit time is the power P , the electric power provided to the circuit is

$$P = \frac{(\Delta q)V}{\Delta t} = \underbrace{\frac{\Delta q}{\Delta t}}_{\text{Current } I} V$$

The term $(\Delta q)/(\Delta t)$ is the charge per unit time or the current I in the circuit. Thus, the electric power is the product of current and voltage.

■ **ELECTRIC POWER**
 When there is a current I in a circuit as a result of a voltage V , the electric power P delivered to the circuit is

$$P = IV \quad (20.6)$$

SI Unit of Power: watt (W)

Power is measured in watts, and Equation 20.6 indicates, therefore, that the product of an ampere and a volt is equal to a watt.

Many electrical devices are essentially resistors that become hot when provided with sufficient electric power: toasters, irons, space heaters, heating elements on electric stoves, and incandescent light bulbs, to name a few. In such cases, it is possible to obtain two additional, but equivalent, expressions for the power. These two expressions follow directly upon substituting $V = IR$, or equivalently $I = V/R$, into the relation $P = IV$:

$$P = IV \quad (20.6a)$$

$$P = I(IR) = I^2R \quad (20.6b)$$

$$P = \left(\frac{V}{R}\right)V = \frac{V^2}{R} \quad (20.6c)$$

Example 5 deals with the electric power delivered to the bulb of a flashlight.