

Characterization of a Freezing/Melting Transition in a Vibrated and Sheared Granular Medium

Karen E. Daniels & Robert P. Behringer

*Department of Physics and Center for Nonlinear and Complex Systems
Duke University, Durham, NC, 27708, USA*

We describe experiments on monodisperse spherical particles in an annular cell geometry, vibrated from below and sheared from above. This system shows a freezing/melting transition such that under sufficient vibration a crystallized state is observed, which can be melted by sufficient shear. We characterize the hysteretic transition between these two states, and observe features reminiscent of both a jamming transition and critical phenomena.

1 INTRODUCTION

Granular systems are frequently subject to shearing and/or vibration. Shearing a granular system can compact and crystallize it (Tsai et al., 2003); tapping will compact it (Nowak et al., 1997); in thin vibrated layers there can be coexistence of crystallized and disordered states (Prevost et al., 2004); and highly vibrated granular systems become gas-like. By varying the relative strength of shear and vibrational methods of injecting energy, we are able to study the interaction of the two effects.

Without vibration, sheared granular materials undergo a phase transition from solid-like to fluid-like behavior: the particles must become unjammed before they can move. We seek to understand what effects vibrations have on such transitions, and on the characteristics of the states on either side of the transition. This is particularly interesting because granular systems are athermal, and one might naively expect that vibrations would play a temperature-like role.

We perform experiments in a classic geometry, annular Couette-like flow (Savage and Sayed, 1984; Miller et al., 1996; Losert et al., 2000; Mueth et al., 2000), with monodisperse particles. Shear and vibration provide competing effects, with the system evolving to a crystallized state when the kinetic energy provided by the vibration is greater than that provided by the shear. The transition is hysteretic, and fluctuations in the packing fraction and the breadth of the force distribution both become large as the crystallized state is approached, in similarity to phase transitions in other systems.

2 EXPERIMENT

The experimental apparatus consists of an annular region containing monodisperse polypropylene spheres, as shown in Fig. 1, with the pressure and volume set from below by a spring within an electromagnetic shaker. The particles are sheared from above and vibrated from below, while the sidewalls are stationary. A more detailed description of the apparatus is given in Daniels and Behringer (2004). To characterize the states, we obtain high-speed video images of particles at the outer Plexiglas wall, laser position measurements of the bottom plate (cell volume), and force time series from a capacitive sensor flush with the bottom plate. For the experiments described in this paper, we fix the frequency of vibration ($f = 60$ Hz) and number of particles ($N \approx 71200$), and vary the amplitude of vibration A and shear rate Ω . The nondimensionalized control parameters are the peak acceleration $\Gamma \equiv A(2\pi f)^2/g = 0$ to 7, and nondimensional

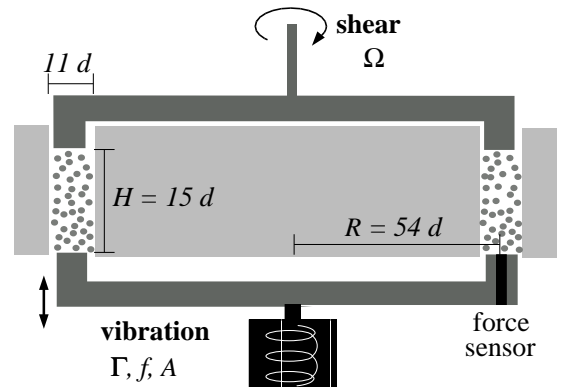


Figure 1: Schematic cross-section of experiment (not to scale).

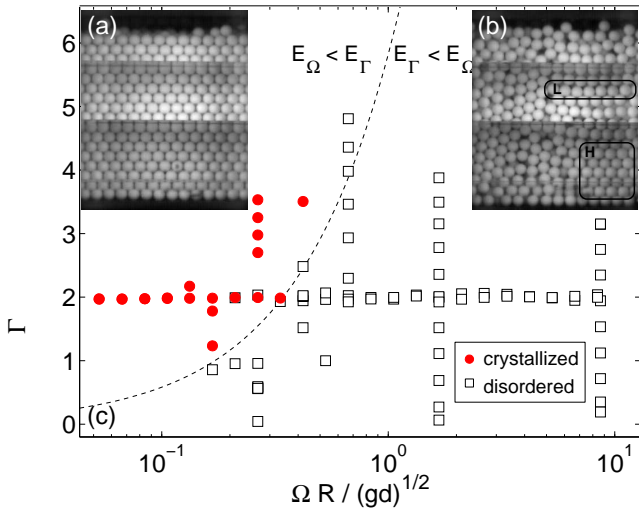


Figure 2: Sample images, viewed from outer wall. (a) Crystallized state: $\Gamma = 2.0$ and $\tilde{\Omega} = 0.078$. (b) Disordered state: $\Gamma = 2.0$ and $\tilde{\Omega} = 0.47$. Linear (L) and hexagonal (H) clusters marked by black boxes. (c) Phase diagram for crystallized and disordered states as a function of $\tilde{\Omega}$ and Γ . Adapted from Daniels and Behringer (2004).

shear rate $\tilde{\Omega} \equiv \Omega R / \sqrt{gd} = 0.058$ to 9.3 .

3 DESCRIPTION OF STATES

In the regime $0 < \tilde{\Omega} < 10$ and $0 < \Gamma < 6$ we observe two distinct granular states of matter: crystallized and disordered. Sample images of these two states are shown in Fig. 2(ab), as viewed from the outer wall. For $\tilde{\Omega} \lesssim 0.1$, the phase boundary between the two states roughly corresponds to a line where the characteristic velocities (and hence the kinetic energies) of the two motions are equal: $\Omega R = 2\pi f A$. Below, we characterize these two states, with further details to be found in Daniels and Behringer (2004).

3.1 Crystallized State

In the solid-like state (see Fig. 2(a)), the balls crystallize into a hexagonally close-packed configuration, here visible only at the outer wall although the order persists across the layer. The contact between the upper layer of the granular material and the shearing wheel is intermittent, with stick-slip motion of the top ~ 2 layers in the manner of Nasuno et al. (1997). The distribution of forces measured at the bottom of the layer is bimodal, and is well-fit by the probability distribution of a sinusoid. This indicates that the material is responding as a solid body moving up and down with the sinusoidal vibrations of the bottom plate.

3.2 Disordered State

In the disordered state, some order remains in the form of hexagonally-packed clusters and linear chains of particles at the outer wall, as marked in Fig. 2(b).

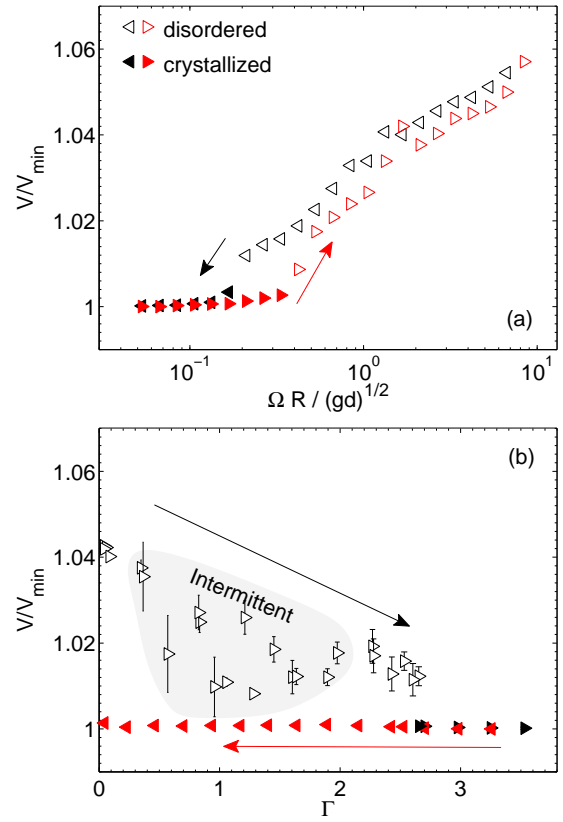


Figure 3: Volume V of cell, scaled by minimum observed volume V_{min} as a function of (a) $\tilde{\Omega}$ and (b) Γ . Triangles point in direction of steps.

For states with $\Omega \gg \Omega_c$, linear chains dominate over hexagonal clusters, with both existing intermittently throughout the disordered regime. The velocity profile extends deeper into the layer than in the crystallized state, with approximately exponential decay. Force distributions measured at the bottom plate show the exponential-like tails characteristic of many granular experiments in disordered, unvibrated granular materials.

For a geometrically similar system, but unvibrated and exposed to a compressional force, shear *ordered* the system into horizontal planes of hexagonal packing, each slipping past the others (Tsai et al., 2003; Tsai and Gollub, 2004). Such a state is different from the 3D crystallized state observed here, in which the layers in the bulk are stationary with respect to each other.

4 TRANSITION

We examine the transition from the disordered to the crystallized state by first preparing a disordered state at high $\tilde{\Omega}$. We then performed two runs, one at constant $\tilde{\Omega} = 0.27$ (starting from $\Gamma = 0$) and the other at constant $\Gamma = 2$ (starting from $\tilde{\Omega} = 8.4$). The mean volume measured for each step of these two runs is shown in Fig. 3.

For steps of decreasing Ω , the system compacts logarithmically until reaching Ω_c , after which the sys-

tem undergoes a first-order phase transition to the crystallized state. Below Ω_c only a small amount of additional compaction occurs, to a state with a volume V_{min} , for which the packing fraction is $\phi = 0.69$. When Ω is increased, the transition back to the disordered state is hysteretic, occurring for $\Omega_h > \Omega_c$.

For steps of increasing Γ , the system also compacts. However, runs approaching the transition are difficult to repeat quantitatively, since there is a great deal of intermittency in the cell volume (see Fig. 3(b) and Daniels and Behringer (2004) for details). For $\Gamma > \Gamma_c$ the system is in the crystallized state. The transition also appears to be first-order, but in this case the hysteresis is so extreme that the experiment was not observed to re-expand when we decreased Γ .

We wish to understand why a crystallized state can disorder by increasing Ω , but not by decreasing Γ . In the case of increasing Ω , the stick-slip behavior in the top layers of the crystallized state is affected by the speed of the upper shearing wheel. As Ω increases, more horizontal momentum is transferred to the upper layer of balls, which results in longer regions of flowing particles. Eventually, the whole layer can be seen to mobilize and the disordering begins to take place throughout the cell. In contrast, for increases in A (and hence Γ) no such increased momentum transfer takes place, and the results are similar to the irreversibility observed for compaction by tapping (Nowak et al., 1997). This transition shows a great deal of similarity to the “freezing-by-heating” transition seen in Helbing et al. (2000), in which individual particles with tunable noise are seen to crystallize as their noise level is increased. Such a system also shows hysteresis in returning to the disordered, mobilized state.

For the run at $\Gamma = 2$, seen in Fig. 3(a) with steps downward in Ω , we observe signatures of the phase transition from disorder to crystallization via both the volume fluctuations and the width of the force distribution, as shown in Fig. 4. As $\Omega \rightarrow \Omega_c$ from above, both the volume fluctuations (measured from the variance of $V(t)$) and the breadth of the force distribution (measured by the kurtosis, or fourth scaled moment, of $F(t)$ on the force sensor) become large.

In granular systems, Edwards and coworkers (Edwards and Oakeshott, 1989) have introduced a temperature-like measure, the compactivity, defined as $X = (\partial V / \partial S)_N$ by analogy with thermodynamics. The central idea is that lower packing fractions correspond to a greater freedom for particle rearrangement, and hence a higher compactivity. In the statistical mechanics of ordinary phase transitions, susceptibilities such as $(\partial^2 A / \partial T^2)_V$ (for free energy A) diverge. For example, the specific heat at constant volume is $C_V = (\partial E / \partial T)_V = -T(\partial^2 A / \partial T^2)_V$. When described in terms of fluctuation-dissipation relations,

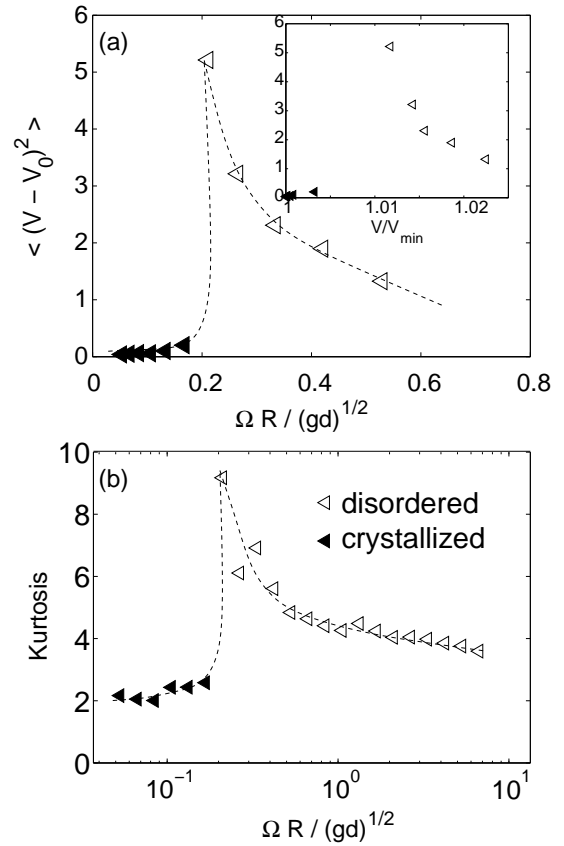


Figure 4: (a) Volume fluctuations as a function of $\tilde{\Omega}$. Inset: Volume fluctuations re-plotted as a function of V_0/V_{min} at each $\tilde{\Omega}$. (b) Force PDF kurtosis as a function of $\tilde{\Omega}$. Both: Lines drawn as guides to the eye.

$k_B T^2 C_V = \langle (E - E_0)^2 \rangle$, where E is the energy of the system and E_0 its mean value. A key result for critical phenomena is that these energy fluctuations (the specific heat) are singular at the critical temperature T_c . Since V has taken the place of E in the Edwards formalism, we should expect increased fluctuations in the volume of the system as we approach X_c , the critical compactivity. In our experiments, volume (and hence X) is set by Ω , and the inset to Fig. 4(a) shows such behavior.

5 DISCUSSION

The two characterizations of transitions we discuss above provide contrasting, but complimentary, information about the nature of the crystallizing phase transition in sheared and vibrated granular materials.

The canonical hallmark of a transition to a jammed/glassy state is the divergence of the viscosity. Glass transitions do not in general contain thermodynamic signatures, such as changes in the volume or specific heat Torquato (2000). For sheared colloids, there are large stress fluctuations near a jamming transition (Lootens et al., 2003), and in simulations of Lennard-Jones particles, force PDFs are observed to broaden (O’Hern et al., 2001; Snoeijer et al., 2004). Similar behavior is observed in this system as well.

Curiously, however, jammed/glassy states are all *disordered*, while the granular system described in this paper makes a transition to a *crystallized* state. In both cases, however, the final states are unable to rearrange.

We observe similarities to critical phenomena in the increased volume fluctuations near the transition, a hallmark at odds with a glass transition. These fluctuations are similar to the density fluctuations observed at the liquid-gas critical point, which occur at diverging length scales. Therefore, further investigations into the nature of this transition should examine what length scales and order parameters are present, including a determination of the sizes of clusters and the spatial correlations between forces.

This work was supported by the NASA microgravity program, grant NNC04GB08G.

REFERENCES

- Daniels, K. E. and Behringer, R. P. (2004). Hysteresis and competition between disorder and crystallization in sheared and vibrated granular flow, *cond-mat/0410087*.
- Edwards, S. F. and Oakeshott, R. B. S. (1989). Theory of powders. *Physica A*, 157(3):1080–1090.
- Helbing, D., Farkas, I. J., and Vicsek, T. (2000). Freezing by heating in a driven mesoscopic system. *Physical Review Letters*, 84:1240–1243.
- Lootens, D., van Damme, H., and Hebraud, P. (2003). Giant stress fluctuations at the jamming transition. *Physical Review Letters*, 90:178301.
- Losert, W., Bocquet, L., Lubensky, T. C., and Gollub, J. P. (2000). Particle dynamics in sheared granular matter. *Physical Review Letters*, 85:1428–1431.
- Miller, B., O’Hern, C., and Behringer, R. P. (1996). Stress fluctuations for continuously sheared granular materials. *Physical Review Letters*, 77:3110–3113.
- Mueth, D. M., Debregeas, G. F., Karczmar, G. S., Eng, P. J., Nagel, S. R., and Jaeger, H. M. (2000). Signatures of granular microstructure in dense shear flows. *Nature*, 406:385–389.
- Nasuno, S., Kudrolli, A., and Gollub, J. P. (1997). Friction in granular layers: Hysteresis and precursors. *Physical Review Letters*, 79:949–952.
- Nowak, E. R., Knight, J. B., Povinelli, M. L., Jaeger, H. M., and Nagel, S. R. (1997). Reversibility and irreversibility in the packing of vibrated granular material. *Powder Technology*, 94:79–83.
- O’Hern, C. S., Langer, S. A., Liu, A. J., and Nagel, S. R. (2001). Force distributions near jamming and glass transitions. *Physical Review Letters*, 86:111–114.
- Prevost, A., Melby, P., Egolf, D. A., and Urbach, J. S. (2004). Nonequilibrium two-phase coexistence in a confined granular layer. *Physical Review E*, 70:050301.
- Savage, S. B. and Sayed, M. (1984). Stresses developed by dry cohesionless granular materials sheared in an annular shear cell. *Journal Of Fluid Mechanics*, 142:391–430.
- Snoeijer, J. H., Vlugt, T. J. H., van Hecke, M., and van Saarloos, W. (2004). Force network ensemble: A new approach to static granular matter. *Physical Review Letters*, 92:054302.
- Torquato, S. (2000). Glass transition – hard knock for thermodynamics. *Nature*, 405:521–522.
- Tsai, J. C. and Gollub, J. P. (2004). Slowly sheared dense granular flows: Crystallization and nonunique final states. *Physical Review E*, 70:031303.
- Tsai, J. C., Voth, G. A., and Gollub, J. P. (2003). Internal granular dynamics, shear-induced crystallization, and compaction steps. *Physical Review Letters*, 91:064301.