Physics 52 **Final** **December 17, 1997**

Name ______________________________

Work directly on these pages and show your work clearly. Properly labeled figures are important and will figure into the grading. *Some basic equations are given on the last page.*

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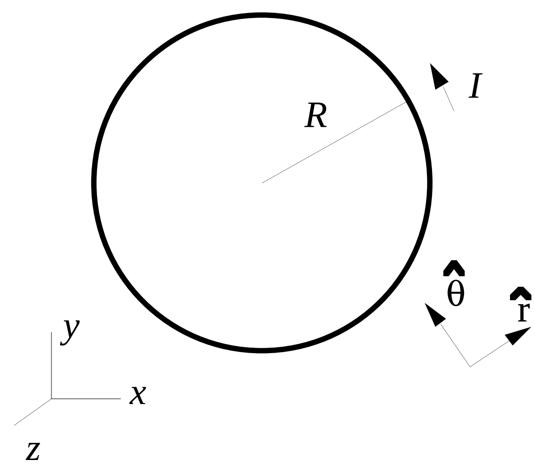
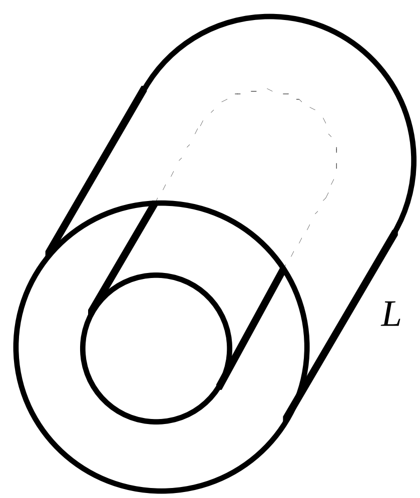
1. [*25 pts*] Find an expression for the total charge Q on a dielectric sphere of radius R if the charge density within the sphere increases with radius according to the expression $\rho(r) = \rho_0\, r/R$.

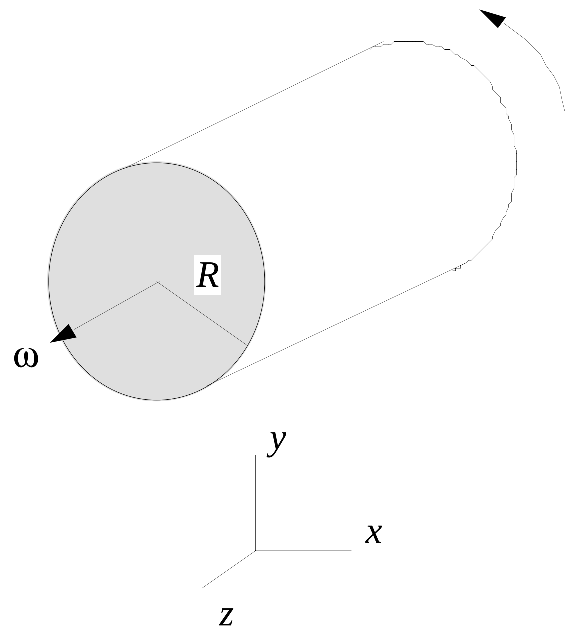
2. [*25 pts*] Write an expression (DO THE INTEGRAL) for the magnetic field $\vec{B}$ at center of a loop of wire carrying a constant current I as shown in the figure. I expect you to not only know the correct direction for the magnetic field, but to use the unit vectors shown and treat them properly so the signs also work out in the algebra. Make sure your algebra is clear on this point.



3. [*25 pts*] Find the capacitance ($C = Q/V$) of two thin conducting cylindrical shells of length L when the inner shell has a radius R_1, and the outer one has a radius R_2. The length L is much greater than either radius so you can ignore end effects. Note that there are several steps to this problem, but you should start by assuming a charge separation and using Gauss' law to determine the electric field between the two shells. You ***need not show*** the zero parts of the closed loop integral, ***but do*** draw end and side views to clearly show me the labeled Gaussian surface you are using.

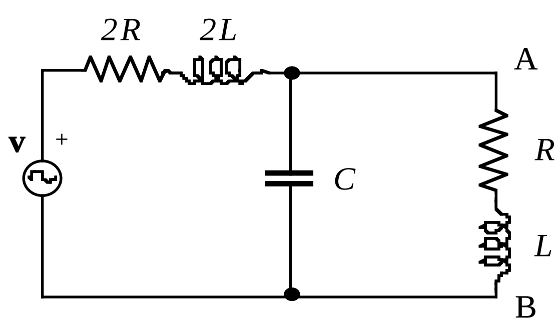


4. [*25 pts*] A long solid dielectric cylinder of radius R carries a uniform charge density ρ throughout its volume. If the cylinder is rotating about it's axis with angular velocity ω as shown in the figure, use Ampere's law to determine the magnetic field $\mathbf{B}(r)$ at a distance $r < R$ from the center of the cylinder. *Hint: This arrangement has some things in common with a coil of wire (solonoid), and the proper Amperean curve is a rectangle. Remember that all of the charge goes past a particular angular position in one period.*



To make it clear that you know what you are doing, I strongly recommend that you draw additional right angle pictures showing the end view of the cylinder and the side view of the cylinder. Show your Amperean curve on these figures and define $\vec{dl}$ and the functional and vector dependence of $\vec{B}$ before you start.

5. [*25 pts*] (a) Label the current in each branch of the circuit shown and write a set of complex equations in terms of **v**, L, C, R, and ω that would allow you to solve for these currents. DO NOT SOLVE THESE EQUATIONS



b) In terms of one of your labeled currents, write a complex expression for the voltage difference $\mathbf{v}_A - \mathbf{v}_B$.

(c) Plot your current and $\mathbf{v}_{AB}$ on the complex plane under the assumption that $\omega L = \sqrt{3}R$. State by what angle, $\mathbf{v}_{AB}$ leads or lags behind your current.

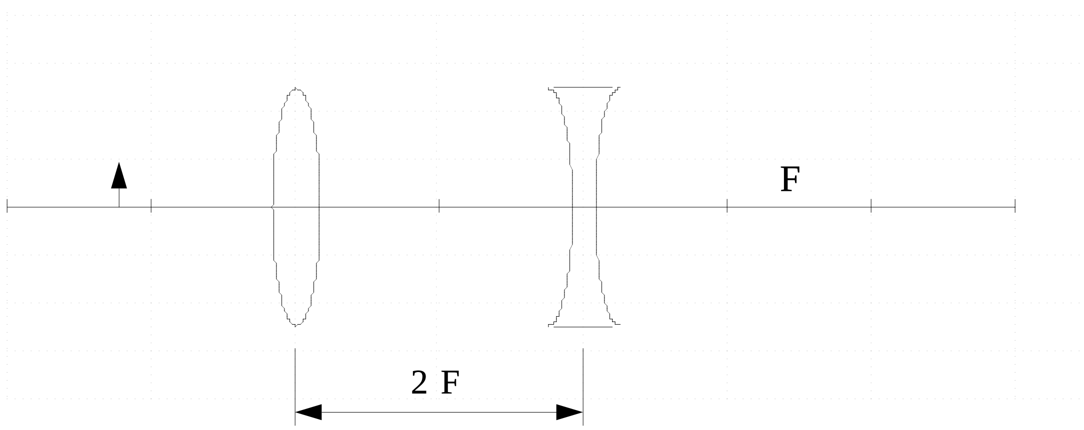
6. [*25 pts*] On a cloudy day with no wind blowing, a fish looks up from the bottom of a large lake of depth D and sees a bright circular area. If the index of refraction of the water is n and that of air is one, find an expression for the area of the surface that is bright. You can not use small angle approximations for this problem, but you should have ***no trig functions*** in your answer.

7. [*25 pts*] The two lenses shown have focal lengths of the same magnitude F, but the left one is converging and the right is diverging. The lenses are located a distance $2F$ apart. The object is at a distance $5F/4$ from the first lens.

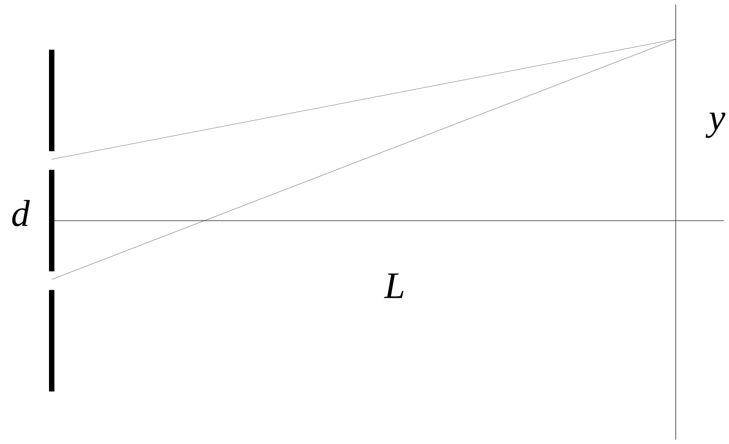
(a) Sketch the three principal rays that can be used to make the intermediate image from the first lens and indicate the ray which is also a principal ray for the second lens.

(b) Use the lens formula to locate the intermediate and final images. Place this final image on the figure.

(c) Determine the magnification of the final image with respect to the first and indicate whether the image is upright or inverted.



8. [*25 pts*] The figure shows an ordinary double slit arrangement with parallel light of wavelength λ incident from the left. The light coming through each slit has amplitude E_0. Ignoring any diffraction effects (the slits are very, very narrow), find an expression for the intensity ($|E|^2$) as a function of the position on the screen y and the constants given here and on the figure. You can do the complex variable math either graphically or algebraically: it takes about the same amount of work either way but does simplify nicely. I recommend that you first get $I(\delta)$ to be sure of maximum partial credit then take $L >> y$ and use the small angle approximations to get $I(y)$.



Work page and equations

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq\,\vec{\mathbf{r}}}{r^3}$$

$$d\Phi_E = \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}}$$

$$dV = -\vec{\mathbf{E}} \bullet d\vec{\mathbf{l}}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{dq\,\vec{\mathbf{v}}\times\vec{\mathbf{r}}}{r^3}$$

$$d\Phi_B = \vec{\mathbf{B}} \bullet d\vec{\mathbf{A}}$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

$$\mathbf{Z} = j\,\omega L, \quad \mathbf{Z} = \frac{1}{j\,\omega C}$$

$$\oint_S \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q}{\varepsilon_0}$$

$$\oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = -\frac{d}{dt}\int_S \vec{\mathbf{B}} \bullet d\vec{\mathbf{A}}$$

$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{l}} = \mu_0 i + \mu_0\varepsilon_0\frac{d}{dt}\int_S \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}}$$

$$\eta_{\mathbf{E}} = \frac{\varepsilon_0 E^2}{2}, \; \eta_{\mathbf{B}} = \frac{B^2}{2\mu_0} \; \text{joules / m}^3$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} \; \text{joules / m}^2\text{ sec}$$

$$P = \int_S \vec{\mathbf{S}} \bullet d\vec{\mathbf{A}} \; \text{watts}$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$E = E_0 e^{j\,\omega(t - r/c)}$$

$$\lambda f = c$$

$$\omega = 2\pi f$$

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$