

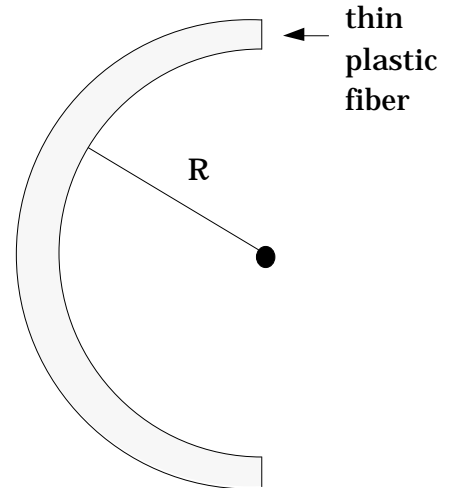
Name _____

Work directly on these pages and show your work clearly. Properly labeled figures are important and will figure into the grading.

Use the proper symbols to show your vectors clearly.

Some basic equations are given at the end of the last page.

1. [30 pts] A thin plastic fiber is shaped into a half circle or radius R as shown in the figure and given a uniform charge per unit length of λ . Determine a fully simplified expression for the electric field at the center (indicated by the solid dot).

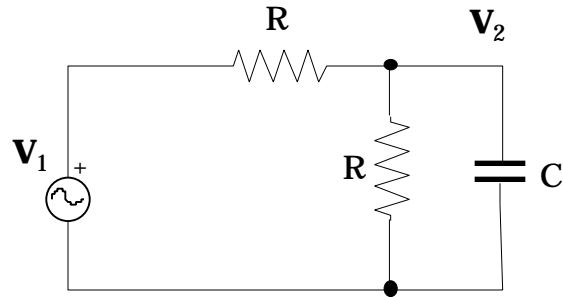


1. (30)	
2. (30)	
3. (25)	
4. (30)	
5. (30)	
6. (25)	
7. (30)	
Total	

2. [30 pts] Two metal train cars, each traveling with speed v , approach each other on a metal railroad track of width w . An upward pointing external magnetic field of strength B exists everywhere. **Each** car has **four** wheels, and **each** wheel has an electrical resistance R at its point of contact with the track rails: the rails and cars are otherwise perfect conductors. **Hint:** The length of the cars does not matter.

- (a) Draw clearly labeled **pictures** that show all elements of this problem.
- (b) Determine an expression for the magnitude and direction of the force on each car. Clearly show everything that's happening on the picture as well as with equations.

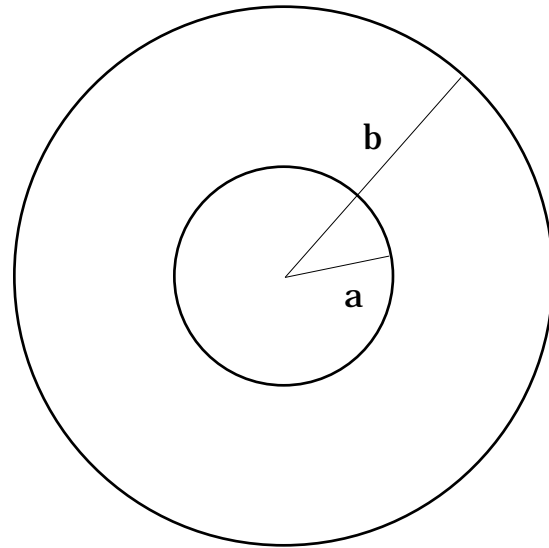
3. [25 pts] (a) For the circuit shown, determine the complex expression for the AC voltage V_2 in terms of the driving EMF V_1 , its angular frequency ω , and the circuit elements.



(b) Evaluate the above expression at the particular frequency where $\omega = 4/RC$, then manipulate this expression so you can make a phasor diagram showing the relative magnitudes and the phase difference between V_1 and V_2 . I want a reasonably accurate picture of this so make a careful drawing.

(c) What is the magnitude of V_2 in terms of the magnitude of V_1 ?

4. [30 pts] Find the capacitance of the two concentric conducting spherical shells shown. To get full credit here I want you to show all of the steps and derive everything starting from Maxwell's equations, $dV = -\mathbf{E} \cdot d\mathbf{r}$, and the definition $C = Q/V$. (If you want to think of this conductor arrangement as a circuit element analogous to the parallel plate capacitor, one wire would be connected through a small hole to the inner shell and the other to the outer shell. This wire would in fact destroy the symmetry of the arrangement, so we either leave it out or neglect its effect.)



5. [30 pts] OK, this is a repeat from the last exam, but it's a neat problem and most of you missed it. Hopefully it'll be a holiday gift this time around.

The magnitude of the magnetic and electric fields inside the coax are

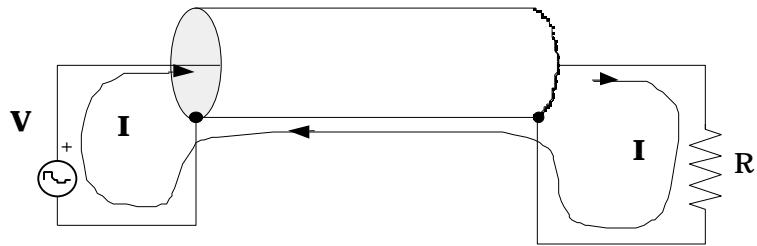
$$B = \frac{\mu_0 i}{2r}, \quad \text{and} \quad E = \frac{V}{r \ln\left(\frac{b}{a}\right)},$$

where a is the radius of the wire and b is the radius of the outer shell. If an infinitesimal part of the power flow inside the coax is given by

$$dP = \vec{S} \cdot d\vec{A},$$

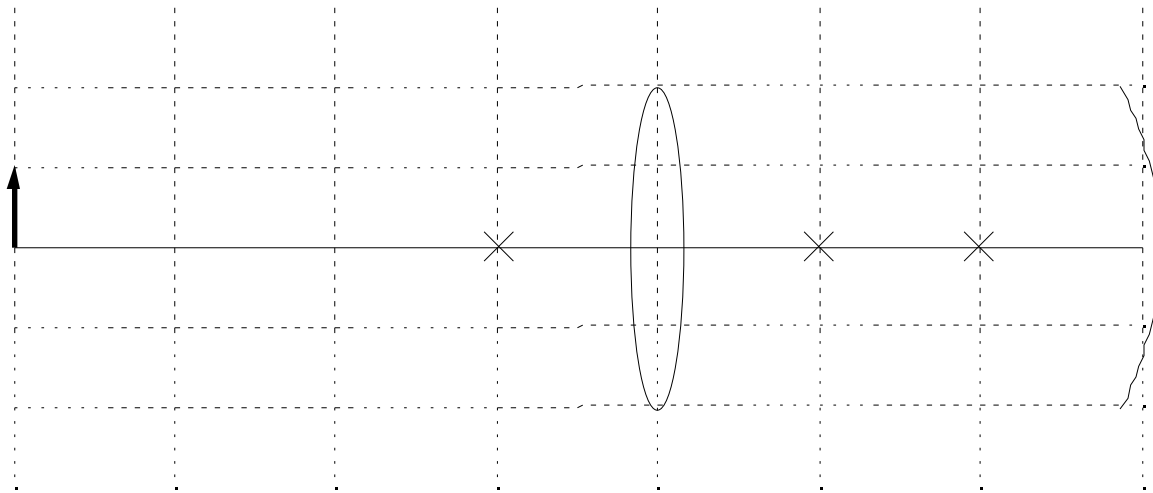
(a) figure out the **directions** of the fields and \vec{S} and **label** them so I know what you mean. You'll need to define an x, y, z and the corresponding r, θ coordinate system.

(b) Identify the correct $d\vec{A}$ on an appropriate drawing of the coax, and **integrate** this expression (all the way) to determine a simplified expression for the power transported by the fields inside the coax.

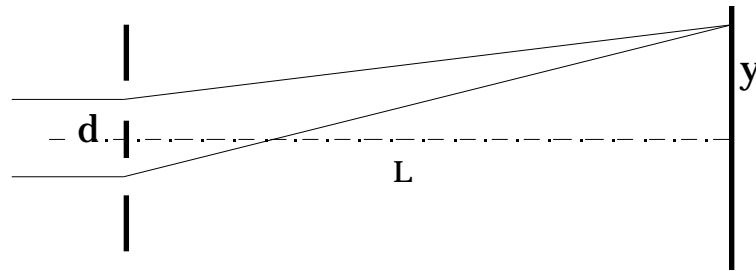


6. [25 pts] An object of height y is located a distance $4F$ to the left of a converging lens and $7F$ to the left of a converging mirror as shown. The lens and mirror both have focal length F and of course the mirror's radius is $2F$. The X's shown on the figure represent the focal points of the lens and mirror, the vertical grid lines are separated by a distance F , and the horizontal ones by a distance y .

This is a several step problem, and I want to see your work, but the only answers I really want are the information about the **final image** seen by an observer positioned to the left of the original object. Where is it, is it erect or inverted, real or virtual, and how big is it compared to y ?



7. [30 pts] Incident light of wavelength λ is incident on a double slit arrangement that is different from the usual in that the upper slit passes a wave with three times the amplitude of the lower one. Assume L is much greater than any other length in the problem and ignore the diffraction effects associated with these slits.



- (a) Starting with the equation for the electric field from the lower slit, $E = E_0 e^{j(t - z/c)}$, write a complex **expression** for the total electric field at a point on the screen. Draw two phasor diagrams: one for the point of maximum intensity and one for the first minimum. What phase shift angle produces the minimum?
- (b) Make the connection between the phase shift just found and the geometry to determine the position y of the minimum on the screen in terms of λ , d , and L .
- (c) Finally, what is the intensity of the light at this minimum compared to the intensity at $y = 0$? Remember $I \propto E^2$.

Work page and equations

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq \hat{r}}{r^2}$$

$$dE = \vec{E} \cdot d\vec{A}$$

$$dV = -\vec{E} \cdot d\vec{r}$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

$$d\vec{B} = \frac{\mu_0}{4} \frac{dq \vec{v} \times \hat{r}}{r^2}$$

$$dB = \vec{B} \cdot d\vec{A}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$Z = jL, \quad Z = \frac{1}{jC}$$

$$\oint_s \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\oint_c \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{A}$$

$$\oint_c \vec{B} \cdot d\vec{r} = \mu_0 i + \mu_0 \epsilon_0 \frac{d}{dt} \int_s \vec{E} \cdot d\vec{A}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}, \quad E = \frac{0E^2}{2}, \quad B = \frac{B^2}{2\mu_0}$$

$$P = \int_s \vec{S} \cdot d\vec{A} \text{ Watts}$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$E = E_0 e^{j(t - z/c)}$$

$$A = 4\pi r^2$$

$$V = \frac{4}{3} \pi r^3$$