Physics 52 **Exam 3** **November 18, 1996**

Name ________________________

Which section? Wed. Rec.____ , Thur. Rec.____

Work directly on these pages and show your work clearly. Properly labeled figures are important and will figure into the grading.

$$\mathbf{Z} = j\omega L, \quad \mathbf{Z} = \frac{1}{j\omega C}$$

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\varepsilon_0}$$

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d}{dt}\int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d}{dt}\int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}, \quad \eta_{\mathbf{E}} = \frac{\varepsilon_0 E^2}{2}, \quad \eta_{\mathbf{B}} = \frac{\mathbf{1}}{2\mu_0}B^2$$

$$P = \int_S \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}$$

1. (20)	
2. (20)	
3. (20)	
4. (20)	
5. (20)	
Total	

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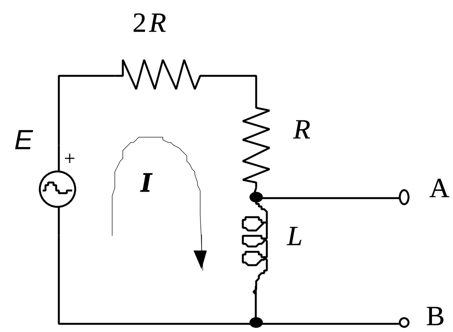
1. [*20 pts*] Use Euler's equation to expand both sides of the expression $e^{j\theta}e^{j\theta} = e^{j2\theta}$, and from the expanded form derive the trig identity for sin 2θ.

2. [*20 pts*] The energy stored in the magnetic field of an inductor can be variously written as $U = \frac{1}{2}LI^2$ or $U = \int \eta_B dV$, where η_B is the energy density in the magnetic field and dV represents a volume element. If the magnitude of the magnetic field outside a long wire is given by $B = \frac{\mu_0 I}{2\pi r}$, determine an expression for the inductance L of a coaxial cable of length D when the radius of the central wire is R_1 and the radius of the surrounding shell is R_2. Assume the current flows on the surface of the inner conductor and returns on the outer conductor.

3. [*20 pts*] (a) Find a complex expression relating the current $\boldsymbol{I}$ to the applied AC voltage **E**. Do not show the time dependence.

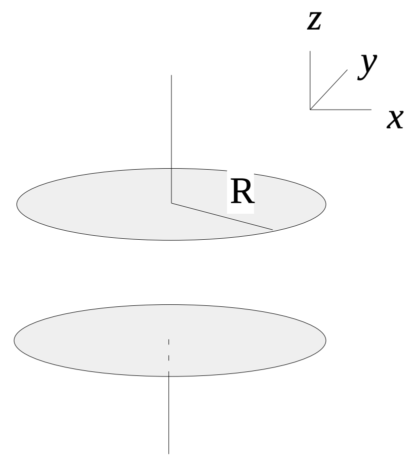


(b) Next determine a complex expression for $\boldsymbol{V}_A - \boldsymbol{V}_B$ in terms of $\boldsymbol{I}$.

(c) At a particularly convinient time in their cycle, plot both voltages and $\boldsymbol{I}$ on the complex plane, and use the plot to find a relationship between w, L, and R such that the phase angle between **E** and $\boldsymbol{V}_{AB}$ will be 45°.

(d) Which of the two voltages leads?

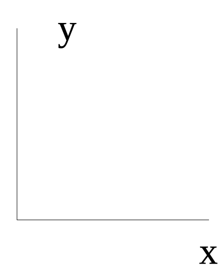
4. [*20 pts*] The electric field is uniform everywhere between the circular plates of the capacitor shown and ***changing*** according to the equation $\vec{\mathbf{E}} = -(E_0 + at)\hat{\mathbf{z}}$ and zero outside the plates. Note the minus sign!



I want you to use the equation $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ to determine an expression for the magnetic field **B** alongside the empty space between the plates at a distance $r > R$ from their axis of symmetry.

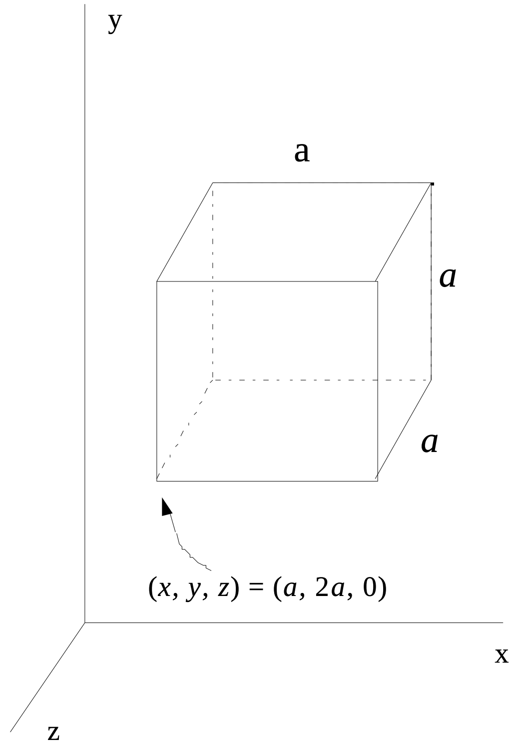
(a) First define the positive directions for **B**, d**l** and d**A** in terms of our standard unit vectors. Sketch these directions on a picture and make sure the last two are consistent with the right hand rule.

(b) Use the integral equation to find an expression for B. Note that **E** does not vary with position inside the plates.



(c) Looking down from a +*z* position, is **B** pointing clockwise or counter-clockwise?

5. [*20 pts*] If the electric field is given everywhere by $\vec{\mathbf{E}} = \frac{E_0}{a} x \hat{\mathbf{z}}$ and the magnetic field by $\vec{\mathbf{B}} = \frac{B_0}{a} x \hat{\mathbf{y}}$, use the Poynting vector to determine the net power flow **out of** the cube shown. Note that your answer may be negative, indicating a power flow into the cube. The sides of the cube have length a, and the front, lower, left corner is positioned at (a, $2a$, 0).



Hint: Only two sides of this cube matter, since the others cancel in pairs.