Name ________________________

Work directly on these pages and show your work clearly.
Work any four of the five 20 point problems.

Mark one answer to each of the following five questions.

1. [*4 pts*] The correct expression for root-mean-squared voltage is

(a) $V_{rms} = \sqrt{\frac{1}{T}\int_t^{t+T} V^2 \, dt}$

(b) $V_{rms} = \left(\frac{1}{T}\int_t^{t+T} \sqrt{V} \, dt\right)^2$

(c) Neither of the above

1 - 5. 6. 7. 8. 9. 10.
Total

2. [4 *pts*] If $\mathbf{V} = (2-3j)\mathbf{I}$, then
(a) the voltage leads the current by more than 45°.
(b) the voltage leads the current by less than 45°.
(c) the current leads the voltage by more than 45°.
(d) the current leads the voltage by less than 45°.

3. [*4 pts*] The correct expression to describe the electric flux passing through a ***flat*** surface is

(a) $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$

(b) $\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$.

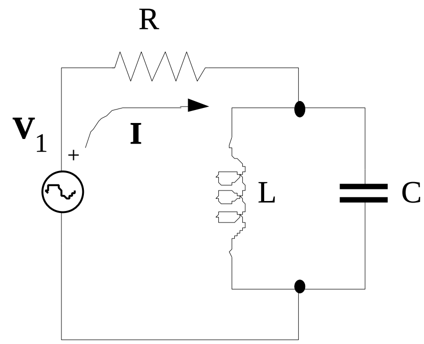
(c) $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$.

(d) $\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$

4. [*4 pts*] When a light ray passes the interface between two transparent materials with different indices of refraction, total internal reflection occurs when
(a) light enters a medium with a larger index of refraction.
(b) light enters a medium with a smaller index of refraction.
(c) the reflected light is parallel to the surface.
(d) the refracted light is parallel to the surface.

5. [*4 pts*] The Poynting vector at the curved surface of a cylindrical resistor points
(a) out of the resistor.
(b) into the resistor.
(c) along the surface of the resistor.

6. [*20 pts*] If the frequency of the source $\mathbf{V}_1$ is ω,



(a) write a simplified complex expression for the combined L and C impedance shown in this figure. What I want here is the equivalent impedance that could replace these two elements and convert the circuit into one with a single loop. Simplified means no compound fractions and j appearing only in $j\omega$ combinations.

(b) Determine a simplified complex expression for the current through the resistor in terms of R, L, C, ω, and $\mathbf{V}_1$. This is best done using the result from (a).

(c) If you've done this right, there should be an obvious frequency ω_0 where the current is zero. What is the phase relationship between **I** and $\mathbf{V}_1$ at a frequency just below this ω_0? Make a sketch showing **I** and $\mathbf{V}_1$ at this this frequence (slightly less than ω_0). This sketch should roughly indicate the correct relative phase and amplitude between this current and voltage.

7. [*20 pts*] The equation for an electromagnetic wave traveling in the positive z-direction is

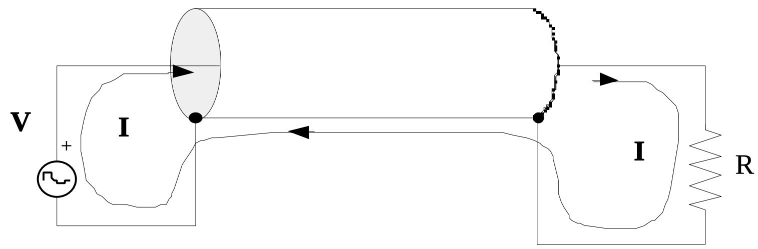
$$E = E_0 \cos 2\pi f\left(t - \frac{z}{c}\right) .$$

If the wave originates at an FM radio station broadcasting at a frequency of 10^8 oscillations/second (100 MHz), plot E against position *z* at a particular time $t = 0$. Start your plot at z = 0 and extend it for at least one full cycle of the wave. Label your graph to show the scale on the *z* axis, and determine the distance in centimeters between the positive peaks of this wave.

There are no tricks here except to pick easy points on the wave that you can evaluate without the aid of a calculator. Remember that the velocity of light is 3×10^8 m/second, and that $\cos(-\theta) = \cos(\theta)$.

8. [*20 pts*] The magnitude of the magnetic field inside the coax shown is

$$B = \frac{\mu_0 i}{2\pi r},$$



and the magnitude of the electric field (you can derive this some other time) is

$$E = \frac{V}{r \ln\left(\frac{b}{a}\right)},$$

where a is the radius of the wire and b is the radius of the outer shell. **Note:** The wire inside the coax and the cylindrical shield are both equipotential surfaces, and their potential difference is V.

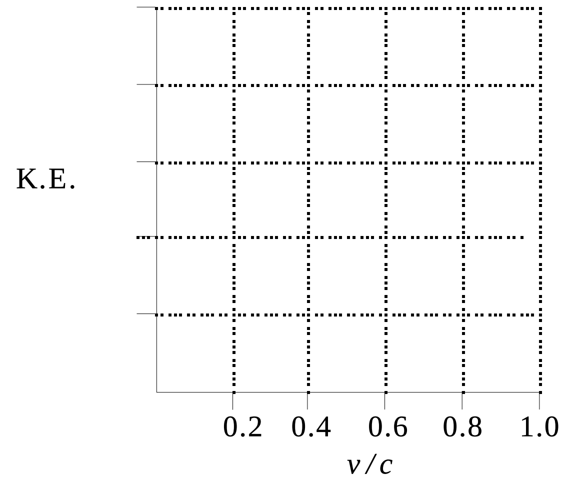
If an infinitesimal part of the power flow inside the coax is given by

$$dP = \vec{\mathbf{S}} \quad d\vec{\mathbf{A}},$$

(a) figure out the ***directions*** of the fields and $\vec{\mathbf{S}}$ and ***label*** them so I know what you mean. You'll need to define an x, y, z and the corresponding r, θ coordinate system.

(b) Identify the correct $d\vec{\mathbf{A}}$ on an appropriate drawing of the coax, and ***integrate*** this expression (all the way) to determine a simplified expression for the power transported by the fields inside the coax.

9. [*20 pts*] (a) If a particle of mass M is traveling at 0.2c (γ 1.02), what is its kinetic energy in terms of M and c? The kinetic energy is still just the energy associated with the motion of the particle, i.e., it does not include the particle's rest energy. For this to be interesting, you should also realize it is the energy that must be expended to accelerate the particle to this velocity. One of the equations on the last page will prove useful.



(b) If it's traveling twice as fast at 0.4c (γ 1.09), what is its kinetic energy?

(c) To double the speed again so it's traveling at 0.8c (γ 1.7), how much energy must we expend?

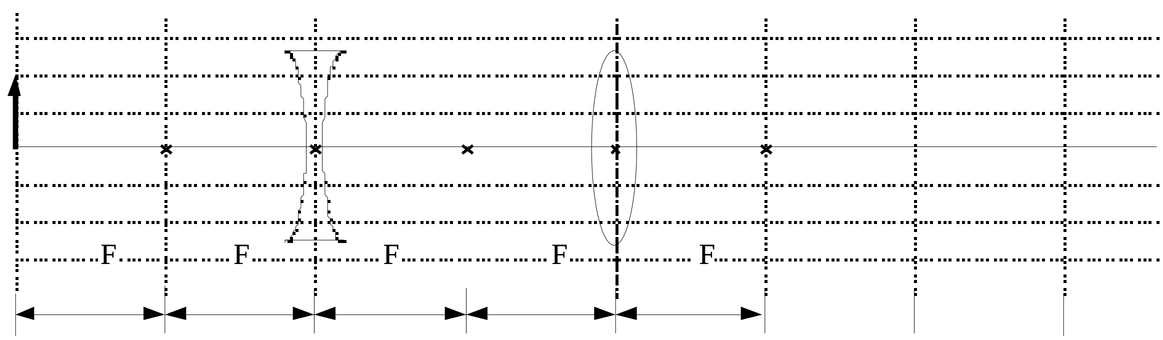
(d) Make a rough plot of these points on the graph provided.

10. [*20 pts*] A diverging lens and a converging lens both have focal lengths whose magnitudes are F. The lenses are separated by a distance $2F$ as shown with an object y located a distance $2F$ to the left of the diverging lens.

(a) From the top of the arrow ***draw*** three rays that are principal rays for the first lens. One of these rays is also a principal ray of the converging lens. ***Indicate*** this ray clearly and ***display*** its behavior in passing through the converging lens.

(b) Is the ***intermediate*** image formed by the diverging lens real or virtual, erect or inverted?

(c) Use the thin lens equation twice to determine simplified expressions for the position of the ***intermediate*** and ***final*** images with respect to the ***converging*** lens. Your answer will be in terms of F. Indicate the approximate position and orientation of the final image on the figure.



Work page

$$\mathbf{Z} = j\omega L, \quad \mathbf{Z} = \frac{1}{j\omega C}$$

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\varepsilon_0}$$

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{d}{dt}\int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}} = \mu_0 i + \mu_0\varepsilon_0 \frac{d}{dt}\int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}, \quad \eta_{\mathbf{E}} = \frac{\varepsilon_0 E^2}{2}, \quad \eta_{\mathbf{B}} = \frac{\mathbf{1}}{2\mu_0}B^2$$

$$P = \int_S \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} \text{ with units Watts / m}^2$$

$$E^2 = P^2c^2 + M^2c^4$$

$$E = \gamma mc^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$