

Name \_\_\_\_\_

Which section? Wed. Rec.\_\_\_\_ , Thur. Rec.\_\_\_\_

*Work directly on these pages and show your work clearly. Properly labeled figures are important and will figure into the grading.*

*Perform the integrations this time, unless the problem specifically asks for just the integral expression.*

**Mark one answer to each of the following five questions.**

1. [4 pts] In the equation  $d\mathbf{F} = i\mathbf{dr} \times \mathbf{B}$ ,  $d\mathbf{r}$  is an infinitesimal element of
- (a) the path followed by the current.
  - (b) the line from the current to the point where  $\mathbf{B}$  is to be determined.
  - (c) an imaginary closed curve encircling the current.

2. [4 pts] In the equation  $d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{i\mathbf{dl} \times \hat{\mathbf{r}}}{r^2}$ ,  $d\mathbf{l}$  is an infinitesimal element of
- (a) the path followed by the current.
  - (b) the line from the current to the point where  $\mathbf{B}$  is to be determined.
  - (c) an imaginary closed curve encircling the current.
  - (d) an imaginary closed curve along  $\mathbf{B}$ .

3. [4 pts] In the equation  $\oint_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_C$ ,  $d\mathbf{r}$  is an infinitesimal element of
- (a) the path followed by the current.
  - (b) an imaginary closed curve encircling the current  $I_C$ .
  - (c) an imaginary closed curve along  $\mathbf{B}$ .

4. [4 pts] In the equation  $\xi = \oint_C \mathbf{E} \cdot d\mathbf{r} = -\frac{d\phi_B}{dt}$ ,  $d\mathbf{r}$  is an infinitesimal element of
- (a) an imaginary closed curve along  $\mathbf{B}$ .
  - (b) an arbitrary imaginary closed curve.
  - (c) an imaginary closed curve encircling some current.

5. [4 pts] Lenz's law says that the induced current will be in a direction
- (a) opposite to  $\mathbf{B}$ .
  - (b) opposite to  $d\mathbf{B}/dt$ .
  - (c) that produces a magnetic field to oppose  $\mathbf{B}$ .
  - (d) that produces a magnetic field to oppose  $d\mathbf{B}/dt$ .

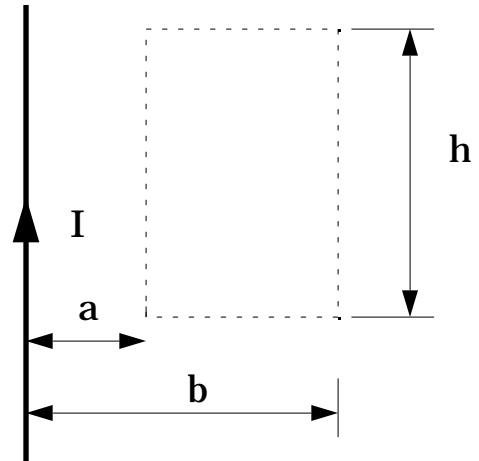
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
Total

6. [10 pts] Two parallel wires are carrying current in opposite directions as shown.

Sketch the magnetic field due to each wire (don't sum them), and indicate the **direction** of the force on each wire.

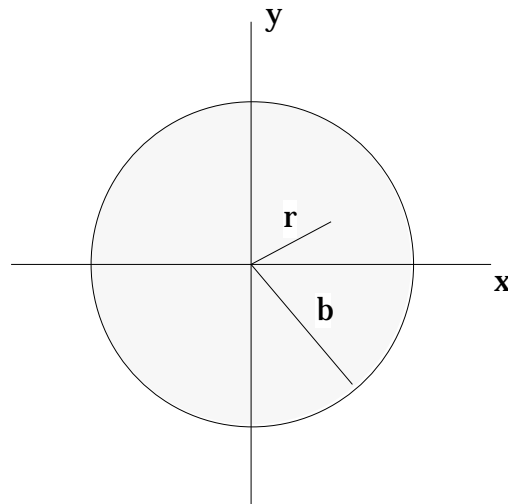
 $I_1$  $I_2$ 

7. [15 pts] If the current carrying wire and the dashed rectangle are both in the plane of the paper, determine an expression for the total flux passing through the rectangle. The magnitude of the magnetic field around a wire is given by  $B = \frac{\mu_0 I}{2\pi r}$ .



8. [15 pts] (a) Use ampere's law to determine the **magnitude** of the magnetic field  $\mathbf{B}$  a distance  $r$  from the axis of a long straight wire carrying total current  $I$  into the paper. Do the case where  $r$  is less than the radius of the wire.

(b) Now use the right hand rule and write  $\mathbf{B}$  in terms of the unit vector  $\hat{\theta}$ . If you haven't already done so, show the direction of the field  $\mathbf{B}$  on the diagram.

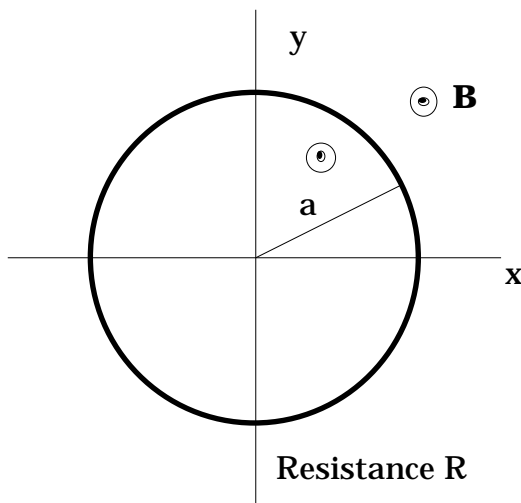


9. [20 pts] A uniform magnetic field  $\mathbf{B}$  points out of the paper as shown and has a magnitude that is increasing with time according to the expression  $B(t) = bt$ . The conducting ring of radius  $a$  and total resistance  $R$  is in the plane of the paper.

(a) Determine an expression for the current induced in the ring at an arbitrary time  $t > 0$ . Be sure and indicate the direction of the current.

(b) Determine an expression for the force on an infinitesimal element of the loop located in the first quadrant in terms of one or more of the unit vectors  $\hat{\mathbf{r}}$  and  $\hat{\boldsymbol{\theta}}$ . Be sure and also show the direction of this force on the figure.

(c) Integrate the  $y$  component of this force to determine an expression for the total force  $\mathbf{F}$  acting on the top half of the loop. Your answer can be left in terms of  $I$  so you only need the direction from part (a).



10. [20 pts] You are given the expressions  $\mathbf{B} = \frac{\mu_0 IN}{2\pi r} \hat{\theta}$  for the field inside a toroid of square cross section and  $\eta_B = \frac{B^2}{2\mu_0}$  for the energy density (defined by  $\eta_B = \frac{dU_B}{dV}$ ). Note that  $B^2$  is just  $\mathbf{B} \cdot \mathbf{B}$ .

The problem is to find a one dimensional integral expression for the energy  $U_B$  stored in the magnetic field of a toroid of rectangular cross section like we used in recitation. As indicated on the drawing, the current  $I$  flows in coils wrapped around the toroidal frame,. The toroid is complete, not truncated to the half donut shown.

(a) The first step is to identify an infinitesimal *volume* within which  $B$  (not  $\mathbf{B}$ ) is constant. But  $dV = dx dy dz$  won't do. I want one which has only a single infinitesimal element. Show and/or describe this infinitesimal volume clearly.

(b) Now write an integral expression for the total stored magnetic energy  $U_B$  complete with limits.

