Limits on the Time Delay Induced by Slow-Light Propagation

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Abstract: We show that there are no fundamental limits to the maximum time delay that can be achieved for pulses propagating through slow-light media, thus suggesting the importance of slow-light methods for practical applications.

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Methods for controlling the propagation velocity of light pulses through material systems [1] hold great promise for important applications. Early work in this area demonstrated that extremely slow group velocities ($v_g << c$) and even superluminal velocities ($v_g > c$ or v_g negative) can be obtained. More recently, interest has turned to the use of slow- and fast-light methods for various applications, many of which require that a pulse of light be delayed by many times the pulse duration in a tunable and controllable fashion. Specific applications in the field of high-speed all-optical signal processing that might benefit significantly from such controllable optical delay lines include random-access memory, network buffering, data synchronization, and pattern correlation.

It has not been clear what physical processes, if any, can lead to a limitation on the total delay that a pulse can experience. For instance, the largest fractional time delay (that is, time delay measured in pulse lengths) reported to date appears to be the value of approximately four reported by Kasapi *et al.* [2]. Here we report the results of a theoretical study of processes that could limit the total time delay. We conclude that, while these processes can impose severe practical limitations, there is no fundamental limit to how large the time delay can become. In brief summary, the argument we present is as follows. One limitation to the overall time delay is imposed by absorption effects, which can limit the distance a light pulse can travel through a material medium. However, absorption can be eliminated through use of electromagnetically induced transparency and related effects. Other limitations are those imposed by group velocity dispersion and spectral reshaping of the pulse. However, we find by explicit calculation that these effects can be rendered negligible by ensuring that the pulse spectrum is considerably narrower than the width of the transparency window. In this contribution we also present numerical results which show that a time delay of 75 pulse lengths is achievable under realistic laboratory conditions.

The time delay (the group delay) experienced by an optical pulse in passing through a material system of length L is given by

$$T_g = \frac{L}{v_g} = \frac{L n_g}{c} \qquad \text{where} \qquad n_g = n + \omega \frac{dn}{d\omega}. \tag{1}$$

Here n_g is the group index and n is the conventional (phase) refractive index. It is useful to introduce the material contribution to the group delay $T_{\rm del} = T_g - L/c$, which is the difference between the group delay and the delay experienced upon propagation through vacuum. This quantity is given by

$$T_{\rm del} = \frac{L}{c}(n_g - 1). \tag{2}$$

Equation (2) shows that the maximal time delay is determined by the group index and by the maximum possible value $L_{\rm max}$ of the propagation distance L through the material medium. This maximum distance can be limited by physical processes such as absorption and diffraction effects. However, techniques such as EIT [4] or coherent population oscillations (CPO) [5–7] are often used to minimize the material absorption. Diffraction effects can also limit the effective value of $L_{\max}$ to the Rayleigh range of the laser light. However, diffraction can be eliminated entirely by working in optical fibers or other guided-wave structures.

There are other potential limitations to the time delay imposed by the spectral variation of the optical properties of the material. Since a pulse necessarily has a non-vanishing spectral width, these effects are intrinsic to the propagation of pulses through a slow-light medium. We consider the propagation of a pulse whose frequency is close to that of a transparency window, such as that created by EIT or CPO. For the present, we assume that the shape of transparency window corresponds to a Lorentzian-shaped dip in the absorption profile, so that the absorption coefficient of this material can be described by the expression

$$\alpha(\delta) = \alpha_0 \left(1 - \frac{f}{1+\delta^2/\gamma^2}\right) \approx \alpha_0 \left[(1-f) - f\frac{\delta^2}{\gamma^2}\right], \tag{3}$$

where α_0 is the value of the background absorption, $\delta = \omega - \omega_0$ is the detuning of the optical frequency ω from the resonance frequency ω_0, and γ is the linewidth of the transparency window. We use the second (approximate) form, which is reasonably reliable for $\delta < \gamma$. In these equations, f is a parameter that describes the depth of the transparency window; complete transparency at line center occurs for $f = 1$. According to the Kramers-Kronig relations, there will be a contribution to the refractive index associated with this absorption feature so that

$$n(\delta) = n_0 + f\left(\frac{\alpha_0\lambda}{4\pi}\right)\frac{\delta/\gamma}{1+\delta^2/\gamma^2} \approx n_0 + f\left(\frac{\alpha_0\lambda}{4\pi}\right)\frac{\delta}{\gamma}\left(1-\frac{\delta^2}{\gamma^2}\right), \tag{4}$$

where n_0 is the background index; under most situations of interest the contribution of n_0 to the group index is very much smaller than that of the second term and will be dropped from the ensuing analysis. From the definition (1) of the group index, we immediately find that

$$n_g \approx f\left(\frac{\alpha_0\lambda}{4\pi}\right)\frac{\omega}{\gamma}\left(1-\frac{3\delta^2}{\gamma^2}\right). \tag{5}$$

We then find that the material delay of Eq. (2) normalized by the pulse length T_0 is given by

$$\frac{T_{\mathrm{del}}}{T_0} \approx \frac{f\alpha_0 L}{2\gamma T_0}\left(1-\frac{3\delta^2}{\gamma^2}\right). \tag{6}$$

Let us now examine the physical processes that might limit the maximum value of the fractional delay. One such process is group-velocity dispersion. We see from Eq. (6) that the fractional delay will be different for different frequency components of a spectrally broad pulse. A pulse of duration T_0 will have a frequency spread of the order of $1/T_0$. For a pulse centered on the transparency window, the spread in fractional group delay will be the difference in group delays for $\delta = 0$ and for $\delta \approx 1/T_0$, and is given by

$$\Delta\left(\frac{T_{\mathrm{del}}}{T_0}\right) \approx \frac{3f}{2}\frac{\alpha_0 L}{\gamma^3 T_0^3}. \tag{7}$$

If we restrict the allowed spread in this quantity to a value of unity (that is, the pulse is allowed to broaden by no more than a factor of two), we find that the length of the interaction region is limited to a maximum value of $L_{\max} = 2\gamma^3 T_0^3/3f\alpha_0$. Through use of Eq. (6), we find that the fractional delay is then limited to

$$\left(\frac{T_{\mathrm{del}}}{T_0}\right)_{\max} = \frac{1}{3}\gamma^2 T_0^2. \tag{8}$$

Note that there is no limit on how large the quantity γT_0 can become. Indeed, one would usually want the pulse duration T_0 to be long compared to $1/\gamma$ so that the entire spectrum of the pulse fits within the transparency window.

Another potential limiting process is the spectral reshaping of the incident pulse due to the frequency dependence of the material absorption [3, 8]. To treat this effect mathematically, we assume that the pulse has a Gaussian spectrum such that $A(\delta) = A_0 \exp[-\delta^2 T_0^2/2]$. After propagating through the medium, the pulse spectrum will be given approximately by

$$A(\delta) = A_0 e^{-(1/2)\delta^2 T_0^2} e^{-f\alpha_0(\delta^2/\gamma^2)L} e^{ikL}, \tag{9}$$

where $k = (\omega/c)[n_0 + f(\alpha_0\lambda/4\pi)(\delta/\gamma)]$. Such a pulse will have a duration T given by

$$T^2 = T_0^2 + f\alpha_0 L/\gamma^2. \tag{10}$$

If, as above, we allow the pulse to broaden by no more than a factor of two, we find that $L_{\rm max} = 3T_0^2\gamma^2/(2f\alpha_0)$. By using this value in Eq. (6), we find that the maximum normalized delay is given by

$$\left(\frac{T_{\rm del}}{T_0}\right)_{\rm max} = \frac{3}{2}\gamma T_0. \tag{11}$$

As noted above, the quantity γT_0 is necessarily greater than unity. Thus, Eq. (11) constitutes a more restrictive condition than does Eq. (8). Since the quantity γT_0 possesses no obvious physical upper bound, this treatment shows that arbitrarily long fractional time delays should be achievable. Note, however, that to achieve the delay given by Eq. (11), it is necessary that the medium possess a reasonably large optical depth (before saturation) given by $\alpha_0 L = (4/3)(T_{\rm del}/T_0)^2_{\rm max}$.

To illustrate these points, we show in Fig. 1 the results of a numerical simulation of pulse propagation though a slow-light medium. The simulation was performed by solving the reduced wave equation using a Fourier transform method for an optical response given by Eqs. (3) and (5). In this example, a pulse is delayed by 75 pulse lengths under realistic laboratory conditions. The pulse undergoes some attenuation and some broadening, but the overall integrity of the pulse is well preserved.

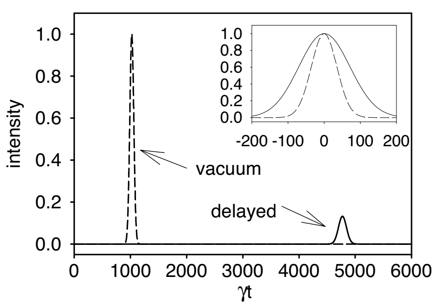

FIG. 1: Numerical simulation demonstrating a large pulse delay in a slow-light medium. The intensity evolution of a Gaussian pulse emerging from the medium for the case of vacuum (dashed line) and a slow light medium (solid line) with $\alpha_0 L = 7500$, $1-f = 8\times10^{-5}$, and $\gamma T_0 = 50$ is shown. The relative time delay is $T_{\rm del}/T_0 = 75$, as predicted by Eq. (11). The inset shows the vacuum and delayed pulses overlaid so that their peaks coincide; it is seen that the delayed pulse is approximately twice as wide and remains highly symmetric.

In summary, we have developed simple physical arguments which suggest that there is no fundamental limit to the fractional time delay a pulse can experience in passing through a slow-light medium. The limited pulse delays observed to date are thus the consequence of practical difficulties, not of fundamental issues.

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