Numerical study of slow light via stimulated Brillouin scattering in optical fibers

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Abstract: We study numerically Stokes pulse propagation in a continuous-wave-pumped Brillouin fiber amplifier. Time delay and pulse broadening of the Stokes pulse are studied in the small-signal and saturation regimes.

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There is a great deal of recent interest in slow light, where the group velocity υ_g of a pulse is much less than the speed of light in vacuum c. Controllable slow light can be used in applications such as optical buffering, variable true time delay and optical information processing, and can be achieved using the large dispersion associated with a resonance of a material system. Electromagnetically induced transparency [1] and coherent population oscillations [2] are two techniques that make use of a narrow transparency induced at the center of an absorbing resonance by an intense coupling laser field to achieve a large group index $n_g = n + \omega(dn/d\omega)$ and a small group velocity $\upsilon_g = c/n_g$. One can also use an amplifying resonance to obtain large dispersion and achieve slow light. Stimulated scattering processes, such as stimulated Raman scattering (SRS) [3] and stimulated Brillouin scattering (SBS), that give rise to amplifying resonances and large dispersion are therefore possible options. Unlike an atomic resonance, an amplifying resonance via SRS or SBS can be created at any wavelength by simply changing the pump wavelength. The use of a single-mode optical fiber as the medium for SRS or SBS may offer additional advantages such as a low pump-power requirement due to long interaction lengths and small mode areas, and the availability of pumping sources at fiber communication wavelengths.

In this paper, we analyze the slow-light effect due to SBS in single-mode optical fibers both analytically and numerically. Our results also show that pulse advancement can be achieved in the gain saturation regime (superluminous pulse propagation). This analysis compliments the recent experimental demonstration of slow light in an optical fiber [4].

In single-mode optical fibers, SBS can be described by 1-dimensional coupled wave equations involving a forward pump wave, a backward Stokes wave and a forward acoustic wave [5]

$$\frac{\partial E_p}{\partial z} + \frac{n}{c}\frac{\partial E_p}{\partial t} = -\frac{\alpha}{2}E_p + ig_2E_s\rho,\ -\frac{\partial E_s}{\partial z} + \frac{n}{c}\frac{\partial E_s}{\partial t} = -\frac{\alpha}{2}E_s + ig_2E_p\rho^*,\ \frac{\partial \rho}{\partial t} + (\frac{\Gamma_B}{2} - i\Delta\omega)\rho = ig_1E_pE_s^*, \qquad (1)$$

where E_p, E_s, and ρ are the amplitudes of the pump wave, Stokes wave, and acoustic wave, respectively, n is the group index of the fiber, α is the loss coefficient of the fiber, $\Gamma_B/2\pi$ is the FWHM of SBS gain, $\Delta\omega = (\omega_{p0} - \Omega_B) - \omega_{s0} \equiv \omega_0 - \omega_{s0}$ is the detuning from the SBS gain line-center ω_0, Ω_B is the SBS frequency shift, ω_{p0} (ω_{s0}) is the center angular frequency of the pump (Stokes) wave, g_1 and g_2 are the coupling constants.

In the condition of a non-depleted CW (continuous-wave) pump, Eqs. (1) can be simplified to give the slowing-varying-envelope-approximation wave equation for the Stokes wave in frequency domain, which is given by

$$\frac{\partial \tilde{E}_s}{\partial z} = \frac{\alpha}{2}\tilde{E}_s - i(\omega - \omega_{s0})\frac{n}{c}\tilde{E}_s - \frac{\frac{1}{2}g_0I_p}{1 - i2(\omega - \omega_{p0} + \Omega_B)/\Gamma_B}\tilde{E}_s. \qquad (2)$$

From Eq. (2), it is seen that the effective index for the Stokes wave is given by

$$\tilde{n}_s(\omega) = n - i\frac{c}{2\omega}\frac{g_0I_p}{1 - i2\delta\omega/\Gamma_B}, \qquad (3)$$

where $\delta\omega = \omega - \omega_0$, I_p is the pump intensity, and $g_0 = 8g_1g_2/(\varepsilon_0 nc\Gamma_B)$ is the line-center SBS gain factor.

From Eq. (3), it is seen that the Stokes wave experiences gain and dispersion in the form of a Lorentzian-shaped resonance that is induced by the pump beam via the SBS process. The gain coefficient $g_s = -2(\omega/c)\mathrm{Im}(\tilde{n}_s)$, refractive index $n_s = \mathrm{Re}(\tilde{n}_s)$, and group index $n_g = n_s + \omega(dn_s/d\omega)$ are given respectively by

$$g_s(\omega) = \frac{g_0 I_p}{1+4\delta\omega^2/\Gamma_B^2}, \quad n_s(\omega) = n + \frac{cg_0 I_p}{\omega}\frac{\delta\omega/\Gamma_B}{1+4\delta\omega^2/\Gamma_B^2}, \quad n_g(\omega) = n + \frac{cg_0 I_p}{\Gamma_B}\frac{1-4\delta\omega^2/\Gamma_B^2}{(1+4\delta\omega^2/\Gamma_B^2)^2}. \tag{4}$$

As shown in Fig. 1, associated with the gain resonance is large normal dispersion ($dn_s/d\omega > 0$) which gives rise to a large n_g and therefore a low υ_g.

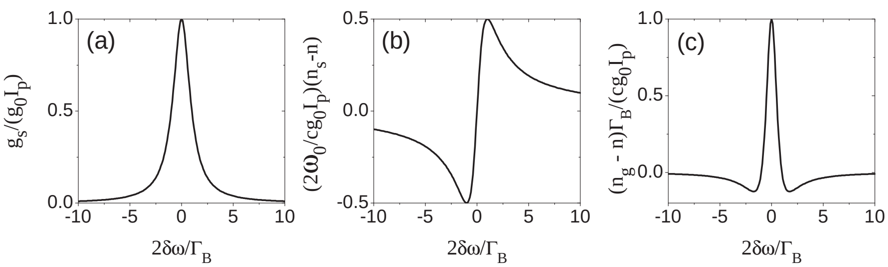

Fig. 1. (a) Gain, (b) refractive index, and (c) group index experienced by the Stokes wave.

Using expression (3) and following the general study on time delay in slow-light media by Boyd *et al.* [6], we obtain the time delay T_d of a Stokes pulse (defined as the difference between the transit times of the pulse with and without SBS)

$$T_d \equiv \frac{L}{c}(n_g - n) = \frac{G}{\Gamma_B}\frac{1-4\delta\omega^2/\Gamma_B^2}{(1+4\delta\omega^2/\Gamma_B^2)^2} \simeq \frac{G}{\Gamma_B}(1-12\delta\omega^2/\Gamma_B^2) \quad \text{when} \quad 4\delta\omega^2/\Gamma_\mathrm{B}^2 \ll 1, \tag{5}$$

where $G = g_0 I_p L$, L is the length of the fiber. When $\delta\omega$ = 0, $T_d = G/\Gamma_B$. For a long Gaussian-shaped Stokes pulse, the pulse emerging from the optical fiber is also Gaussian-shaped with a longer pulse length, where the pulse width broadening factor B_t is given by

$$B_t \equiv T_\mathrm{FWHM,out}/T_\mathrm{FWHM,in} = \left[1 + 16\ln(2)G/(T^2_\mathrm{FWHM,in}\Gamma_B^2)\right]^{1/2}. \tag{6}$$

Equations (5) and (6) indicate that both the time delay and pulse broadening increase with gain. For a short-duration input pulse, both T_d and B_t deviate from the analytic predictions, and the output pulse is distorted due to higher-order dispersion (especially 3rd-order dispersion), as will be shown in the numerical simulations.

Although the small-signal analysis provides useful insight into the slow-light effect by SBS in optical fibers, it is not applicable to general conditions such as pulsed pump, pump-depletion, or short Stokes pulses. In this case, one has to solve Eqs. (1) numerically. We study the slow Stokes pulse propagation in single-mode optical fibers by numerically solving Eqs. (1) using the method of characteristics [7]. In our numerical simulations, we assume that the pump is CW, the Gaussian Stokes pulse is on the SBS line center ($\delta\omega$ = 0) and has no frequency chirping, and use the following parameters L = 50 m, λ = 1550 nm, n = 1.45, effective mode area A_{eff} = 50 μm^2, α = 0.2 dB/km, $\Gamma_B/2\pi$ = 40 MHz, and g_0 = 5 $\times 10^{-11}$ m/W.

Figure 2 shows the time delay T_d and pulse broadening factor B_t as a function of gain coefficient G for a relatively long pulse of FWHM of 120 ns (its FWHM bandwidth $\sim$ 3.7 MHz is much smaller that the SBS gain bandwidth of 40 MHz). The input pulse peak power is 0.1 μW and G is limited up to the SBS threshold of $\sim$ 25. As can be seen in Fig. 2(a), T_d increases linearly with G and agrees well with the analytical prediction (dashed line) by Eq. (5) when G is small ($\lesssim$8). At larger G, T_d first increases slowly with G and then reaches its maximum before deceasing with G. This behavior can be explained by gain-saturation-induced pulse advancement because the pulse is large enough to deplete the pump and the leading edge of the pulse experiences higher amplification than the trailing edge [8]. Gain saturation will thus limit the maximum time delay a Stokes pulse can experience. The calculated broadening factor also agrees well with the analytic result (dashed line) given by Eq. (6) when G is small, as shown in Fig. 2(b). At larger G, the numerical result deviates from the analytic result because of gain saturation. In the saturation regime, the pulse

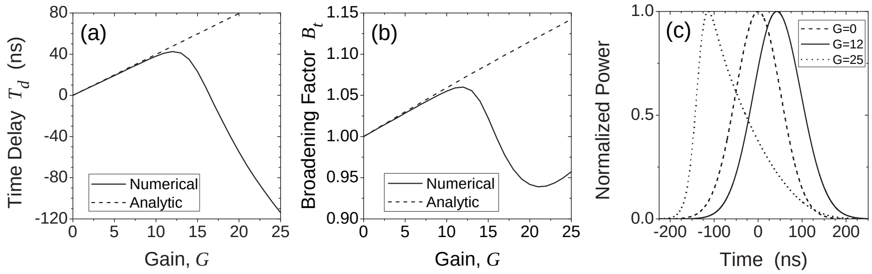

Fig. 2. (a) Time delay vs. gain. (b) Broadening factor vs. gain. (c) Normalized output pulses at G = 0, 12, and 25 (the output peak powers are 9.977×10^{-8} W, 1.301×10^{-2} W and 0.8778W, respectively). The input Stokes pulse has a peak power of 0.1 μW and a FWHM width of 120 ns. The dashed lines in (a) and (b) are obtained from Eq. (5) and Eq. (6), respectively.

width even narrows due to pulse distortion. Figure 2(c) shows the normalized output pulse shapes at G = 0, 12, and 25, respectively. Evidently, the output pulse with a maximum time delay ($\sim$ 43 ns at G = 12) has little distortion, whereas the output pulse is advanced by $\sim$ 114 ns in the highly saturated regime (G = 25), but is distorted significantly.

For the purpose of comparison, we show in Fig. 3 the scenario for a relatively short pulse of 20 ns (FWHM) and of the same peak power power of 0.1 μW. Compared to the 120-ns pulse, the 20-ns pulse reaches the saturation regime at a larger G and the maximum time delay is accordingly larger. At the same time, however, pulse broadening is significant due to the broader bandwidth of the pulse. The variations of T_d and B_t with G deviate significantly from the analytic results (shown as dashed lines) even when G is small. In Fig. 3(c), we plot the normalized output Stokes pulses at G = 0 and 16, illustrating small pulse distortion at a maximum delay of $\sim$ 49 ns at G = 16.

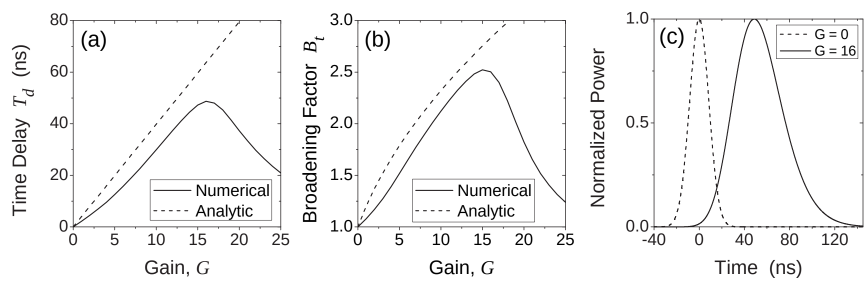

Fig. 3. (a) Time delay vs. gain. (b) Broadening factor vs. gain. (c) Normalized output pulse powers at G = 0 and 16 (the output peak powers are 9.977×10^{-8} W and 9.985×10^{-2} W, respectively). The input Stokes pulse has a peak power of 0.1 μW and a FWHM width of 20 ns. The dashed lines in (a) and (b) are obtained from Eq. (5) and Eq. (6), respectively.

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