

## Mesoscopic fluctuations in quantum dots in the Kondo regime

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(Received 3 June 2003; revised manuscript received 8 August 2003; published 14 October 2003)

Properties of the Kondo effect in quantum dots depend sensitively on the coupling parameters and so on the realization of the quantum dot—the Kondo temperature itself becomes a mesoscopic quantity. Assuming chaotic dynamics in the dot, we use random matrix theory to calculate the distribution of both the Kondo temperature and the conductance in the Coulomb blockade regime. We study two experimentally relevant cases: leads with single channels and leads with many channels. In the single-channel case, the distribution of the conductance is very wide as  $T_K$  fluctuates on a logarithmic scale. As the number of channels increases, there is a slow crossover to a self-averaging regime.

DOI: 10.1103/PhysRevB.68.161305

PACS number(s): 73.23.Hk, 72.15.Qm, 73.63.Kv

Advances in fabrication of nanoscale devices have made possible unprecedented control over their properties.<sup>1</sup> While the transport properties of quantum dots—systems of electrons confined to small regions of space—have been studied extensively, it is only recently that their many-body aspects have been probed<sup>2,3</sup> using the exquisite control now available. On the other hand, a continually fascinating aspect of nanophysics is the presence of quantum coherence and the interference “fluctuations” that it engenders.<sup>1,4</sup> It is natural, then, to ask how this classic mesoscopic physics effect influences the newly found many-body aspects.

Quantum dots are usually formed in one of two ways: either by depleting a two-dimensional electron gas at the interface of a GaAs/AlGaAs heterostructure—a *semiconductor* quantum dot (SQD) (Ref. 1)—or by attaching leads to ultrasmall *metallic* grains (MQD).<sup>2</sup> If the dot is weakly coupled to all leads used to probe it, there is an energy price to be paid for an excess electron to enter the dot, simply because of the energy required to localize the charge. This regime is known as the Coulomb blockade (CB). In a SQD, one attains this regime by reducing the number of propagating channels in the leads; after the threshold of the lowest transverse mode becomes larger than the Fermi energy in the leads, an effective 1D tunneling barrier is formed in this lowest mode. In a MQD, in contrast, tunneling junctions with the leads can be made by oxidizing the surface of the dot. The main difference for our purposes is that in the case of a MQD there are many channels propagating at  $E_F$  whereas in SQD's there is only one.

By capacitively coupling to a metallic gate, one can control the dot's potential, allowing current to flow and bringing out mesoscopic effects.<sup>1</sup> For certain values of the gate voltage  $V_g$ , the electrostatic energy difference between  $N$  and  $N+1$  electrons in the dot is balanced by the interaction with the gate: an electron can freely jump from the left lead onto the dot and then into the other lead. This produces a peak in the conductance at this  $V_g$ . By changing the gate voltage, one can observe large peaks followed by valleys. Both the peak heights and peak spacings fluctuate as one varies  $V_g$ . The distribution and correlations of the peak heights and spacings have been studied experimentally, as well as theoretically using random matrix theory (RMT).<sup>4</sup> In general, there is agreement between experiment and theory.<sup>4–6</sup>

Whether the temperature  $T$  is larger or smaller than the typical width of the resonances,  $\Gamma$  has a large effect on the dot's conductance. As long as  $T \gg \Gamma$ , transport proceeds by either resonant tunneling (peaks) or the off-resonant process known as cotunneling (valleys).<sup>7</sup> However, when  $T$  is sufficiently small many-body effects become important; in particular, at  $T=0$  if  $N$  is odd the conductance can be of the order of the conductance quantum  $e^2/h$  in the Coulomb blockade valleys where naively one expects a strong suppression of the conductance.<sup>8,9</sup> Only recently have experiments actually seen this enhancement.<sup>10–13</sup>

The temperature scale at which these many-body effects develop is called the Kondo temperature  $T_K$ .<sup>14</sup> This scale is exponentially sensitive to the tunneling rates from the leads to the dot. Since these tunneling rates depend on the wave functions of the dot,  $T_K$  shows mesoscopic fluctuations.

Experimentally, mesoscopic fluctuations of the Kondo effect in SQD's is observed. Perhaps the clearest example is in Ref. 12 where the conductance as a function of magnetic field in the Kondo regime is shown. In addition, the frequent observation of the Kondo effect in certain valleys but not others<sup>11</sup> suggests that mesoscopics plays a role. Likewise, the observation of the Kondo effect in two adjacent valleys<sup>13</sup> argues for the important role of fluctuations.<sup>3</sup>

At low temperatures,  $T_K$  is the only important energy scale,<sup>14</sup> and thus one can calculate the distribution of any property given the distribution of  $T_K$ . We will focus on the conductance as it is the most relevant experimentally.

Assuming chaotic dynamics in the dot, we use random matrix theory to calculate the distribution of  $T_K$  in the CB valleys. We restrict our attention to the case of odd  $N$  and  $S=1/2$  and study how the fluctuations depend on the number of propagating channels. We go on to calculate the distribution of the conductance at  $T=0$  and discuss the effect of finite temperature.

*The Hamiltonian.* The Hamiltonian of the system,  $\hat{H} = \hat{H}_{\text{dot}} + \hat{H}_{\text{leads}} + \hat{H}_T$ , consists of the quantum dot's Hamiltonian, the Hamiltonian of the leads, and the tunneling Hamiltonian which describes dot-lead couplings.  $\hat{H}_{\text{dot}}$  has both a one-body part and an interaction part. Impurities and/or scattering from the boundaries are taken into account statistically by using a random matrix model for the one-body Hamiltonian. The most important interactions of the

electrons on the dot are the charging effect and exchange.<sup>7</sup> Thus, we find

$$\hat{H}_{\text{dot}} = \sum_{j\sigma} \varepsilon_j \hat{c}_{j\sigma}^\dagger \hat{c}_{j\sigma} + E_C (\hat{n} - \mathcal{N})^2 - J_S \hat{S}^2, \quad (1)$$

where  $\varepsilon_j$  is the single-particle energy spectrum on the dot;  $\mathcal{N}$  is the dimensionless gate voltage used to tune the number of electrons on the dot;  $\hat{n}$  is the number operator for excess electrons on the dot;  $\hat{S}$  is the spin operator; and  $J_S$  and  $E_C$  are the exchange constant and charging energy, respectively.

The Hamiltonian of the leads differs in the SQD and MQD cases. In the SQD, a tunneling junction is made by pinching until just after the last propagating mode is cut off—the lead is effectively 1D. In the MQD, the leads are wide and many propagating channels can tunnel through the oxide barrier. In the general case of  $N_L$  and  $N_R$  channels, we can label all the states in the leads by a channel index and 1D momentum,  $\hat{H}_{\text{leads}} = \sum (\varepsilon_k + E_m) \hat{c}_{mk\sigma}^\dagger \hat{c}_{mk\sigma}$ , where  $\varepsilon_k + E_m$  are the one-particle energies in the  $m$ th channel with momentum  $k$ .

Finally, the Hamiltonian for the dot-lead coupling is  $\hat{H}_T = \sum (t_{mj} \hat{c}_{mk\sigma}^\dagger \hat{c}_{j\sigma} + \text{H.c.})$ , where  $t_{mj}$  are the matrix elements for each of the  $N_L + N_R$  quantum channels tunneling into the  $j$ th state of the quantum dot.  $|t_{mj}|^2$  is proportional to the intensity of the wave function  $j$  in the dot. We assume that it is independent of  $k$  in the lead since the typical energy scale for changing wave functions in the clean leads is much larger than other relevant energy scales.

We would like to rewrite this Hamiltonian in terms of energy rather than momentum states in the leads. We define new operators  $\hat{c}_{m\epsilon\sigma}^\dagger = \hat{c}_{mk\sigma}^\dagger |d\epsilon/dk|^{-1/2}$  which create states with energy  $\epsilon = E_m + \hbar^2 k^2/2m$  in the  $m$ th channel. The normalization is chosen so that  $[\hat{c}_k, \hat{c}_{k'}^\dagger] = \delta(k - k')$  implies  $[\hat{c}_\epsilon, \hat{c}_{\epsilon'}^\dagger] = \delta(\epsilon - \epsilon')$ . In terms of these operators, the Hamiltonian is

$$\hat{H}_L = \sum_{m\sigma} \int d\epsilon \epsilon (\hat{c}_{m\epsilon\sigma}^\dagger \hat{c}_{m\epsilon\sigma}), \quad (2)$$

$$H_T = \sum_{jm\sigma} \int d\epsilon [\tilde{t}_{mj} \hat{c}_{m\epsilon\sigma}^\dagger \hat{c}_{j\sigma} + \text{H.c.}], \quad (3)$$

with  $\tilde{t}_{mj} \equiv t_{mj} |d\epsilon/dk|^{1/2}$ . Note that even for the same  $\epsilon$  the derivatives and the lower limits on the integrals will be different for different channels.

*Mapping to single-channel Anderson model.* The Hamiltonian of the system bears close resemblance to an  $N$ -channel impurity problem. However, since we are considering the  $S = 1/2$  odd  $N$  valley Kondo regime,<sup>7</sup> only one of the spatial states in the dots is of consequence, namely the highest energy orbital which we shall refer to as  $j_{\text{res}}$ . When the amplitude from this one level leaks out through the barriers it is intuitively plausible that it couples to only one linear combination of the transverse wave functions in the leads. Hence we look for a rotation in the channel basis that chooses the correct wave function in the leads to couple to level  $j_{\text{res}}$  on

the dot, denoting the new creation and destruction operators in the leads  $\hat{z}^\dagger$  and  $\hat{z}$ . If we choose

$$\hat{z}_{1\epsilon\sigma}^\dagger = \sum_{m \in L} (\tilde{t}_{mj_{\text{res}}} \hat{c}_{m\epsilon\sigma}^\dagger) / t_L, \quad (4)$$

where  $t_L = (\sum_{m \in L} \tilde{t}_{mj_{\text{res}}}^2)^{1/2}$  and similarly choose  $\hat{z}_2^\dagger$  in terms of  $m \in R$ , only one channel couples to the quantum dot on each lead. We may choose the remaining channels in any way just making sure they are orthonormal to  $\hat{z}_{1,2}$ . This defines a unitary rotation matrix  $U$ , the first two rows of which we have specified.<sup>15</sup> The Hamiltonian has now been effectively reduced to two parts: (1) two single-channel leads connected to a quantum dot with tunneling matrix elements  $t_L$  and  $t_R$ , and (2)  $N_L + N_R - 2$  decoupled channels.<sup>16</sup>

The decoupled channels do not contribute, and the solution to the first problem is available in the literature:<sup>8</sup> By making another rotation in  $R$ - $L$  space, one can reduce the problem to a one-channel Kondo problem plus a decoupled channel. Thus all the thermodynamic properties of the quantum dot are those of a one-channel Kondo problem. For the conductance, one must rotate back to the original basis of leads. We may use the Kubo formula to calculate the conductance at finite temperatures with the result<sup>17</sup>

$$G_K = G_0 F_K(T/T_K), \quad (5)$$

$G_0$  is defined in terms of the level widths  $\Gamma_m = 2\pi\nu_m |t_{mj_{\text{res}}}|^2$  by

$$G_0 = 4 \frac{2e^2}{h} \frac{\sum_{i \in L} \Gamma_i \sum_{k \in R} \Gamma_k}{\left( \sum_{i \in L, R} \Gamma_i \right)^2}. \quad (6)$$

$F_K(T/T_K)$  is a universal function which has been calculated<sup>18,19</sup> using the numerical renormalization group technique.

*Fluctuations of  $T_K$ .* Because of the exponential sensitivity of  $T_K$  on the wave functions, fluctuations are expected to be strong in the weak tunneling regime of a SQD. As one increases the number of channels one obtains self-averaging. Our goal is to study the crossover between these two limits.

For a simple Anderson model for the quantum dot, the Kondo temperature from a scaling argument<sup>20</sup> is

$$T_K = \frac{U^2}{\sqrt{\epsilon_0(\epsilon_0 + U)}} \left( \frac{\Gamma}{U} \right)^{1/2} \exp \left[ \frac{\pi}{2} \frac{\epsilon_0(\epsilon_0 + U)}{\Gamma U} \right], \quad (7)$$

where  $\epsilon_0$  is negative and measures the energy difference between  $E_F$  and the singly occupied level on the dot,  $\Gamma$  is the width of the level, and  $U = 2E_C$  is the on-site interaction. In the case of a quantum dot, we have to correct for the fact that the spectrum in the dot is dense, and thus the high-energy cutoff for the Kondo Hamiltonian is not given by  $U$  but by the mean spacing  $\Delta$ .<sup>7,20</sup> Varying  $V_G$  adjusts  $\epsilon_0$ ; expressing this as a fraction of  $U$  by choosing  $\epsilon_0 = -xU (x > 0)$ , we write  $T_K$  as

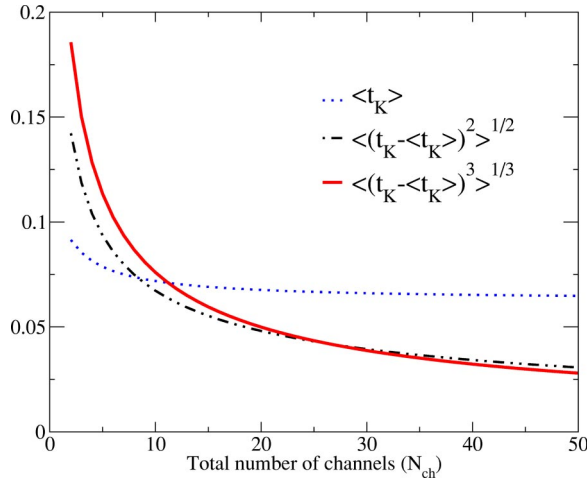


FIG. 1. (Color online) The moments of the distribution of  $T_K$  as a function of the number of channels in the leads  $N_{\text{ch}}$  [Eq. (12)]. The first three moments are shown—average (dotted), root mean square (dot-dashed), and asymmetry (solid). The moments indicate that the distribution is badly behaved for small  $N_{\text{ch}}$ , while it becomes reasonable for  $N_{\text{ch}} > 10$ .

$$T_K = \frac{\Delta}{\sqrt{x(1-x)}} \left( \frac{\Gamma}{U} \right)^{1/2} \exp \left[ -\frac{\pi x(1-x)}{2} \frac{U}{\Gamma} \right]. \quad (8)$$

To calculate the distribution of the Kondo temperatures, one needs an appropriate distribution for  $\Gamma$ . For simplicity, we assume that the different channels in the lead couple to the dot wave function in a similar way on average so that the average level widths for all the channels are the same although they fluctuate independently. We allow, however, for a different number of channels in the left and right leads. From random matrix theory, it is known that the total level width follows a  $\chi^2$  distribution with  $N_{\text{ch}} \equiv \beta(N_L + N_R)$  degrees of freedom:<sup>4</sup>

$$P(\Gamma) = \left( \frac{\Gamma}{2\bar{\Gamma}} \right)^{N_{\text{ch}}/2} \frac{1}{(N_{\text{ch}}/2 - 1)!} \Gamma^{N_{\text{ch}}/2 - 1} e^{-N_{\text{ch}}\Gamma/2\bar{\Gamma}}, \quad (9)$$

where  $\beta = 1(2)$  with(out) time reversal symmetry.

The distribution of  $t_K \equiv T_K \sqrt{x(1-x)}/\Delta$  follows from Eqs. (10) and (11). While we have not been able to obtain a closed-form expression for  $P(t_K)$ , we can calculate all its moments:

$$\langle t_K^n \rangle = \frac{2(aN_{\text{ch}})^{N_{\text{ch}}/2}}{(N_{\text{ch}}/2 - 1)!} \left( \frac{\pi x(1-x)n}{2aN_{\text{ch}}} \right)^{(n+N_{\text{ch}})/4} \times K_{(n+N_{\text{ch}})/2} \left[ \sqrt{2\pi a N_{\text{ch}} x(1-x)} \right], \quad (10)$$

where  $a = U/2\bar{\Gamma}$  and  $K_n(z)$  is the modified Bessel function of the second kind.

Note, first, that when  $P(\Gamma)$  is broad, as for  $N_{\text{ch}} = 1$  or 2, the fluctuations of  $T_K$  are on a logarithmic scale and so are huge. On the other hand, the Kondo temperature depends on only the sum of the decay widths. On adding more channels one gets a sharply peaked function for  $\Gamma$ . Thus, the huge

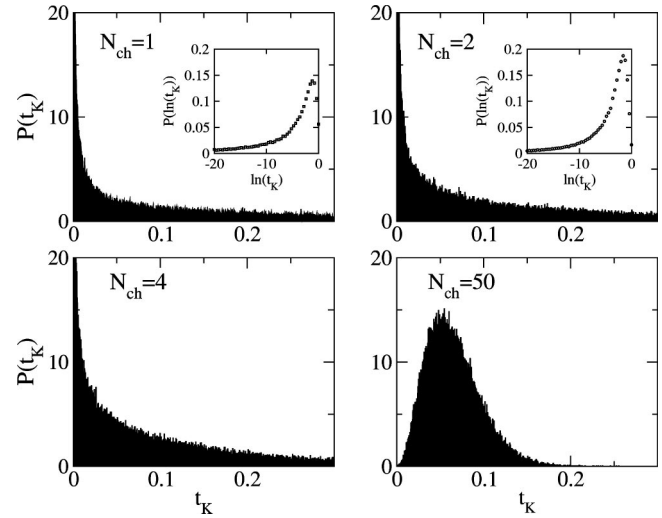


FIG. 2. Probability density of  $T_K$ ,  $P(T_K)$ , calculated numerically for  $\bar{\Gamma} = 0.2U$  and  $x = 1/2$  in four cases:  $N_{\text{ch}} = 1, 2, 4,$  and  $50$ . The main panels are on a linear scale while the insets are the distribution of  $\ln T_K$ . The distributions for small  $N_{\text{ch}}$  are very broad while that for  $N_{\text{ch}} = 50$  is reasonably behaved, though not yet Gaussian. Note that applying a magnetic field doubles the number of effective channels in the leads—changing  $N_{\text{ch}} = 2$  to 4, for instance—and so increases the probability of seeing a Kondo effect.

logarithmic fluctuations inherent in the Kondo effect are reduced until something well behaved is obtained.

In Fig. 1, we have plotted the average, rms, as well as cubic deviation as a function of the number of channels using Eq. (12). The number of channels at which there is a crossover, from logarithmic fluctuations to a distribution well characterized by its mean and variance, is about  $N_{\text{ch}} \approx 10$ . We also note that the cubic moment and rms decay only algebraically and so the crossover to self-averaging is slow—there is no “scale” associated with this crossover. The main features of the dependence on  $N_{\text{ch}}$  are clearly borne out in the numerical results for the full distribution shown in Fig. 2.

*Zero temperature.* In the case of many channels coupled to the quantum dot,  $\langle T_K \rangle$  is a meaningful quantity and we can define a  $T = 0$  regime. To calculate the distribution of conductance at zero temperature, we use  $G_0 = G_K(T=0)$  taken from Eq. (8). This distribution is also a good approximation in the case  $N_L \gg N_R$  as  $T_K$  depends on  $N_L + N_R$  and hence does not fluctuate as much as  $G_0$ .

Using again the random matrix theory result (11) for the level widths, the result in closed form is:

$$P(g) = \frac{1}{N} \frac{k(g)^{N_L/2 - 1} + k(g)^{N_R/2 - 1}}{[k(g) - 1][1 + k(g)]^{[(N_L + N_R)/2] - 3}}, \quad (11)$$

$$k(g) = -1 + \frac{1}{2g} + \frac{1}{2} \sqrt{\frac{1}{g^2} - \frac{4}{g}},$$

where  $g$  is defined by  $G = 4(2e^2/h)g$ .

We have plotted  $P(g)$  in Fig. 3 for three different channel realizations to emphasize that the distribution function can be changed quite dramatically by changing  $N_L$  and  $N_R$ . As

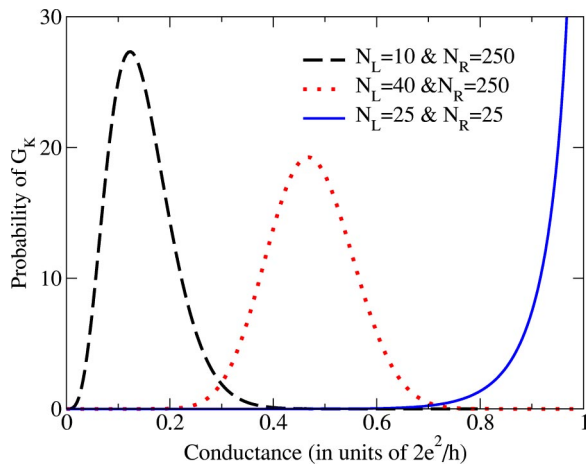


FIG. 3. (Color online) Probability density of the conductance for three different sets of  $(N_L, N_R)$  in the  $T=0$  regime.

expected, in the symmetric case the most probable conductance is  $2(e^2/h)$ . By putting in some asymmetry, we can obtain a variety of distributions. Note, in particular, the highly asymmetric case in which the distribution peaks at a conductance much smaller than  $2(e^2/h)$ . Since the fluctuations of  $T_K$  depend on the sum of  $\Gamma_i$ , we expect in a heavily asymmetric case that the zero-temperature result will show even at finite temperatures. Such a situation may be realized in a metallic particle when one of the “leads” is a scanning tunneling microscope tip.

**Conclusions.** Our main results are (1) the Kondo enhanced conductance is given by Eq. (7) for any number of

channels in the left and right lead; (2) we have calculated the distribution of both  $T_K$  (Figs. 1 and 2) and the prefactor  $G_0$  (Fig. 3); (3) at  $T=0$ , fluctuations are dominated by the prefactor, and Eq. (13) gives the distribution.

Turning to a comparison with the experiments, we first note that most of the experimental dots were not in the deep Coulomb blockade regime,<sup>10–13</sup> and so application of our results is problematic because we have neglected charge fluctuations. Nonetheless, we use our theory to make a few experimentally relevant estimates. One of the difficulties in the experiments is finding several odd valleys in a row which show the Kondo effect.<sup>21</sup> The changing characteristics of the dot as a function of gate voltages contribute to this difficulty, but the mesoscopic fluctuations of  $T_K$  will also play a role. RMT predicts that neighboring energy levels have completely uncorrelated wave functions and so that  $T_K$  in sequential odd  $N$  valleys are uncorrelated. For the dots of Ref. 12, for instance, we estimate that  $T_K > T$  in about 0.4 of the odd valleys (using  $\langle \Gamma/U \rangle = 0.2$  and  $T/\Delta = 0.02$ ).

Fluctuations of  $G_K$  can also be generated by continuous tuning parameters such as magnetic field, revealing a correlation scale. The expected scale is the semiclassical scale for changing chaotic wave functions. Interestingly, in Ref. 12 these fluctuations are correlated on a scale which is an order of magnitude larger. This is consistent with measurements of the correlations in CB valleys in the cotunneling regime.<sup>22</sup> Neither measurement is understood at this time.

We thank P. Brouwer, L. Glazman, K. Matveev, E. Mucciolo, and G. Usaj for useful discussions. This work was supported in part by the NSF (Grant No. DMR-0103003).

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